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Ring Road, New Delhi - 110 029.

Managing Editor : Mahabir Singh
Editor : Anil Ahlawat (BE, MBA)

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editorial

Mathematics and Modelling

Whenever we start any discussion on any topic in mathematics pure or applied without a piece of paper and pen or a black-board, one is helpless. Models are used in physics, physical chemistry or chemical physics or applied mathematics. Whenever you enter even a chemistry professor's room, you find models of various crystals, organic molecules and the rest. To observe, to touch and feel is a great thing. A Maths professor makes a dot on the black-board and goes on explaining anything you want.

Modelling is useful like a walking stick for an old man. However, models dull our imagination. The ability to do things mentally as well as thinking of the models in the mind will release a person from the fetters of physical models. There is another type of modelling. This is by taking simple concrete examples to learn matrices, calculus and so on. By taking simple simultaneous equations with small values for x and y, try to solve by the rules of matrices. Then check the values obtained by solving the simultaneous equations as one does it in smaller classes.

However, in higher classes – in universities, wave mechanics with simple equations is more popular. Simplified approach is different from elementary thinking of learning multiplication and division by sticks and circles as models. Differential equations can be easily taught, as easily as simple calculus, for medical groups. If youngsters walk with walking sticks, the method of walking freely will be difficult. A physical model is like a walking stick, which has to be discarded after the initial stage.

Anil Ahlawat
Editor

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material. During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

Prof. Dr. Ramanaiah Gundala, Former Dean of Science and Humanities, Anna University, Chennai

PROBLEM Set 139

JEE MAIN

1. The natural numbers are written as a sequence of digits 1 2 3 4 5 6 7 8 9 10 11 12 . . . . The 2014^{th} digit in the sequence is

(a) 0 (b) 3

(c) 5 (d) 7

2. The sum of the last 4 digits of the natural number 7^{2014} is

(a) 23 (b) 24

(c) 26 (d) 27

3. If a, b, c, d, p are positive numbers such that

$a + b + c + d + p = 8$

$a^2 + b^2 + c^2 + d^2 + p^2 = 16$

then the maximum value of p is

(a) $\frac{4}{5}$ (b) $\frac{8}{11}$ (c) $\frac{16}{5}$ (d) $\frac{28}{11}$

4. If $I = \int_0^2 \max\{\ln(1+x^2), 1\}\, dx$, then

$I - 2\left[\ln 5 + \sqrt{e-1} - \tan^{-1}\sqrt{e-1} + \tan^{-1} 2\right] =$

(a) 0 (b) 1 (c) 2 (d) 4

5. A man sent 7 letters to his 7 friends. The letters are kept in the addressed envelopes at random. The probability that 3 friends receive correct letters and 4 letters go to wrong destinations is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{3}{32}$ (d) $\frac{5}{64}$

JEE ADVANCED

6. If $A = (0, 1, -2)$, $B = (2, -1, 0)$, $C = (1, 2, 3)$, then direction ratios of bisector of angle BAC are

(a) $-1, 2, 1$ (b) $1, -1, 0$

(c) $1, 1, 1$ (d) $2, -1, 4$

COMPREHENSION

Consider the curves $C_1 : y = f(x)$ and $C_2 : y = g(x)$. $f(1) = 1$, $g(2) = 3$, the tangents to C_1 and C_2 at points with equal abscissae intersect on the y-axis, the normals at points with equal abscissae intersect on the x-axis.

7. $f(2) + g(1) =$

(a) 0 (b) 1 (c) 2 (d) 3

8. $\int_1^2 f(x)\, g(x)\, dx =$

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

INTEGER MATCH

9. Three circles of radii 1, 2, 3 touch each other externally. The radius of the fourth circle which touches all the three circles internally is

MATCHING LIST

10. Let N be the number of 4-digit numbers satisfying the condition in column-I and the sum S of the digits of N is in column-II.

Column-I		Column-II	
P.	digits increase from left to right	1.	3
Q.	digits decrease from left to right	2.	9
R.	digits non decreasing from left to right	3.	12
S.	digits non increasing from left to right	4.	18

	P	Q	R	S
(a)	2	1	4	3
(b)	1	2	3	4
(c)	3	4	1	2
(d)	4	3	2	1

See Solution set of Maths Musing 138 on page no. 71

Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series

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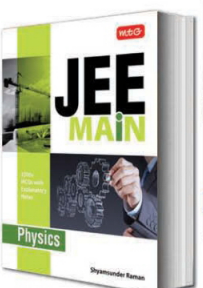

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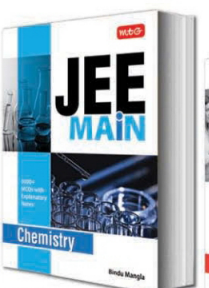

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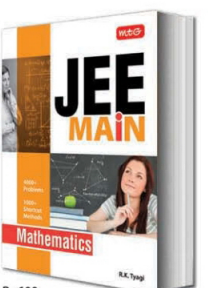

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SOLVED PAPER 2014

JEE Advanced

* ALOK KUMAR, B.Tech, IIT Kanpur

PAPER-1

SECTION-I

One or More Than One Options Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONE or MORE THAN ONE** are correct.

1. Let $f:[a,b]\to[1,\infty)$ be a continuous function and let $g:R\to R$ be defined as

$$g(x)=\begin{cases}0, & \text{if } x<a\\ \int_a^x f(t)dt, & \text{if } a\le x\le b\\ \int_a^b f(t)dt, & \text{if } x>b\end{cases}$$

Then

(a) $g(x)$ is continuous but not differentiable at a

(b) $g(x)$ is differentiable on R

(c) $g(x)$ is continuous but not differentiable at b

(d) $g(x)$ is continuous and differentiable at either a or b but not both

2. For every pair of continuous function $f,g:[0,1]\to R$ such that $\max\{f(x):x\in[0,1]\}=\max\{g(x):x\in[0,1]\}$, the correct statement(s) is(are):

(a) $(f(c))^2+3f(c)=(g(c))^2+3g(c)$ for some $c\in[0,1]$

(b) $(f(c))^2+f(c)=(g(c))^2+3g(c)$ for some $c\in[0,1]$

(c) $(f(c))^2+3f(c)=(g(c))^2+g(c)$ for some $c\in[0,1]$

(d) $(f(c))^2=(g(c))^2$ for some $c\in[0,1]$

3. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(a) the first column of M is the transpose of the second row of M.

(b) the second row of M is the transpose of the first column of M.

(c) M is a diagonal matrix with non-zero entries in the main diagonal.

(d) the product of entries in the main diagonal of M is not the square of an integer.

4. Let $\vec{x},\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\pi/3$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

(a) $\vec{b}=(\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$ (b) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$

(c) $\vec{a}\cdot\vec{b}=-(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$ (d) $\vec{a}=(\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

5. From a point $P(\lambda,\lambda,\lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y=x, z=1$ and $y=-x, z=-1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(a) $\sqrt{2}$ (b) 1 (c) -1 (d) $-\sqrt{2}$

6. Let M and N be two 3×3 matrices such that $MN=NM$. Further, if $M\neq N^2$ and $M^2=N^4$, then

(a) determinant of (M^2+MN^2) is 0

(b) there is a 3×3 non-zero matrix U such that $(M^2+MN^2)U$ is the zero matrix

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

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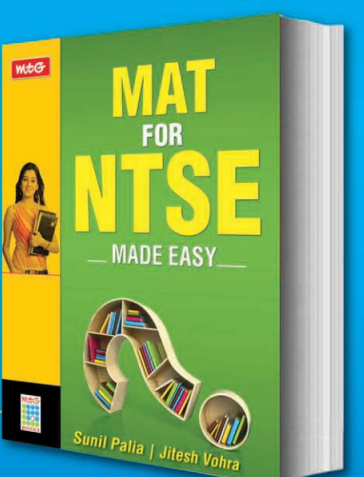

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(c) determinant of $(M^2 + MN^2) \geq 1$
(d) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix.

7. Let $f: (0, \infty) \to R$ be given by $f(x) = \int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$. Then

(a) $f(x)$ is monotonically increasing on $[1, \infty)$
(b) $f(x)$ is monotonically decreasing on $(0, 1)$
(c) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$
(d) $f(2^x)$ is an odd function of x on R

8. Let $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$. Then

(a) $f(x)$ is an odd function
(b) $f(x)$ is a one-one function
(c) $f(x)$ is an onto function
(d) $f(x)$ is an even function

9. A circle S passes through the point $(0, 1)$ and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(a) radius of S is 8 (b) radius of S is 7
(c) centre of S is $(-7, 1)$
(d) centre of S is $(-8, 1)$

10. Let $a \in R$ and let $f: R \to R$ be given by $f(x) = x^5 - 5x + a$. Then

(a) $f(x)$ has three real roots if $a > 4$
(b) $f(x)$ has only one real root if $a > 4$
(c) $f(x)$ has three real roots if $a < -4$
(d) $f(x)$ has three real roots if $-4 < a < 4$

SECTION-II

One Integer Value Correct Type

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive)

11. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point $(1, 3)$ is

12. Let $f: [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10 - x}{10}$ is

13. The largest value of the non-negative integer a for which $\lim_{x \to 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ is

14. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: R \to R$ by $h(x) = \begin{cases} \max\{f(x), g(x)\}, & \text{if } x \leq 0 \\ \min\{f(x), g(x)\}, & \text{if } x > 0 \end{cases}$

The number of points at which $h(x)$ is not differentiable is

15. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is

16. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

17. The value of $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2}(1 - x^2)^5 \right\} dx$ is

18. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\pi/3$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q, and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

19. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2 + a - 14}{a + 1}$ is

20. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

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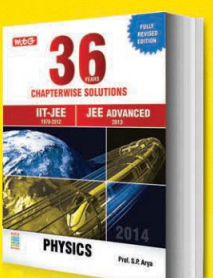

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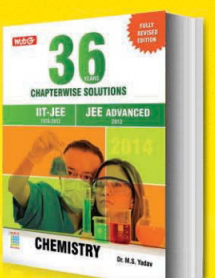

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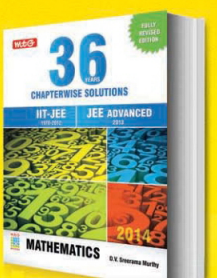

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SECTION-I

Only One Option Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** option is correct.

1. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is
(a) 1/2 (b) 1/3 (c) 2/3 (d) 3/4

2. In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is a third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is
(a) $\frac{3y}{2x(x+c)}$ (b) $\frac{3y}{2c(x+c)}$
(c) $\frac{3y}{4x(x+c)}$ (d) $\frac{3y}{4c(x+c)}$

3. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is
(a) 264 (b) 265 (c) 53 (d) 67

4. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the point P, Q and the parabola at the points R, S. Then the area of the quadrilateral $PQRS$ is
(a) 3 (b) 6 (c) 9 (d) 15

5. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has
(a) only purely imaginary roots
(b) all real roots
(c) two real and two purely imaginary roots
(d) neither real nor purely imaginary roots

6. The following integral $\int_{\pi/4}^{\pi/2} (2\text{cosec}x)^{17}\,dx$ is equal to
(a) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16}\,du$
(b) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17}\,du$
(c) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17}\,du$
(d) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16}\,du$

7. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$. Then $\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx$ is
(a) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (b) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
(c) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

8. Let $f : [0, 2] \to R$ be a function which is continuous on $[0, 2]$ and is differentiable on $(0, 2)$ with $f(0) = 1$. Let $F(x) = \int_0^{x^2} f(\sqrt{t})\,dt$ for $x \in [0, 2]$. If $F'(x) = f'(x)$ for all $x \in (0, 2)$, then $F(2)$ equals
(a) $e^2 - 1$ (b) $e^4 - 1$
(c) $e - 1$ (d) e^4

9. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is
(a) 1051 (b) 1106 (c) 1113 (d) 1120

10. For $x \in (0, \pi)$, the equation
$\sin x + 2\sin 2x - \sin 3x = 3$ has

(a) infinitely many solutions
(b) three solutions
(c) one solution
(d) no solution

SECTION-II

Comprehension Type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data, etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (a), (b), (c) and (d).

Paragraph for questions 11 and 12

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 ; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1, 2, 3$.

11. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\frac{29}{105}$ (b) $\frac{53}{105}$ (c) $\frac{57}{105}$ (d) $\frac{1}{2}$

12. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$ (c) $\frac{11}{105}$ (d) $\frac{7}{105}$

Paragraph for questions 13 and 14

Let a, r, s, t be non-zero real numbers. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point $(2a, 0)$.

13. The value of r is

(a) $-\frac{1}{t}$ (b) $\frac{t^2+1}{t}$ (c) $\frac{1}{t}$ (d) $\frac{t^2-1}{t}$

14. If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(a) $\frac{(t^2+1)^2}{2t^3}$ (b) $\frac{a(t^2+1)^2}{2t^3}$

(c) $\frac{a(t^2+1)^2}{t^3}$ (d) $\frac{a(t^2+2)^2}{t^3}$

Paragraph for questions 15 and 16

Given that for each $a \in (0, 1)$, $\lim_{t \to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$ exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on $(0, 1)$.

15. The value of $g\left(\frac{1}{2}\right)$ is

(a) π (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

16. The value of $g'\left(\frac{1}{2}\right)$ is

(a) $\frac{\pi}{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) 0

SECTION-III

Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (a), (b), (c) and (d), out of which one is correct.

17. Match the following.

	List-I	List-II	
P.	The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)dx = 1$, is	1.	8
Q.	The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	2.	2
R.	$\int_{-2}^{2} \frac{3x^2}{(1+e^x)}\,dx$ equals	3.	4
S.	$\frac{\left(\int_{-1/2}^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right)dx\right)}{\left(\int_{0}^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right)dx\right)}$ equals	4.	0

	P	Q	R	S
(a)	3	2	4	1
(b)	2	3	4	1
(c)	3	2	1	4
(d)	2	3	1	4

18. Match the following.

List-I		List-II	
P.	Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals	1.	1
Q.	Let $A_1, A_2, ..., A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, $k = 1, 2, ..., n$. If $\left\|\sum_{k=1}^{n-1}\left(\overrightarrow{a_k} \times \overrightarrow{a_{k+1}}\right)\right\| = \left\|\sum_{k=1}^{n-1}\left(\overrightarrow{a_k} \cdot \overrightarrow{a_{k+1}}\right)\right\|$, then the minimum value of n is	2.	2
R.	If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is	3.	8
S.	Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is	4.	9

	P	Q	R	S
(a)	4	3	2	1
(b)	2	4	3	1
(c)	4	3	1	2
(d)	2	4	1	3

19. Let $f_1 : R \to R$, $f_2 : [0, \infty) \to R$, $f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x|, & \text{if } x < 0, \\ e^x, & \text{if } x \geq 0; \end{cases}$$

$f_2(x) = x^2$;

$$f_3(x) = \begin{cases} \sin x, & \text{if } x < 0 \\ x \quad , & \text{if } x \geq 0 \end{cases} \text{ and}$$

$$f_4(x) = \begin{cases} f_2(f_1(x)) \quad , & \text{if } x < 0 \\ f_2(f_1(x)) - 1, & \text{if } x \geq 0 \end{cases}$$

List-I		List-II	
P.	f_4 is	1.	onto but not one-one
Q.	f_3 is	2.	neither continuous nor one-one
R.	$f_2 of_1$ is	3.	differentiable but not one-one
S.	f_2 is	4.	continuous and one-one

	P	Q	R	S
(a)	3	1	4	2
(b)	1	3	4	2
(c)	3	1	2	4
(d)	1	3	2	4

20. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, ..., 9$.

List-I		List-II	
P.	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$.	1.	True
Q.	There exists a $k \in \{1, 2, ..., 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers.	2.	False
R.	$\frac{\lvert 1-z_1\rvert\lvert 1-z_2\rvert ... \lvert 1-z_9\rvert}{10}$ equals	3.	1
S.	$1 - \sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals	4.	2

	P	Q	R	S
(a)	1	2	4	3
(b)	2	1	3	4
(c)	1	2	3	4
(d)	2	1	4	3

SOLUTIONS

PAPER-1

1. (a, c): $g(a^-) = 0, g(a^+) = 0$

Also, $g(a^+) = \lim_{h\to 0} \int_a^{a+h} f(t)dt = 0$

Thus g is continuous at $x = a$. Similarly, g is continuous at $x = b$.

$$g'(a^-) = \lim_{h\to 0} \frac{g(a-h)-g(a)}{-h} \text{ (by definition)}$$

$$= \lim_{h\to 0} \frac{0 - \int_a^a f(t)dt}{-h} = \lim_{h\to 0} \frac{0}{-h} = 0$$

$$g'(a^+) = \lim_{h\to 0} \frac{g(a+h)-g(a)}{h} = \lim_{h\to 0} \frac{\int_a^{a+h} f(t)dt - 0}{h}$$

$$= \lim_{h\to 0} \frac{\int_a^{a+h} f(t)dt}{h} = \lim_{h\to 0} \frac{f(a+h)}{1} = f(a) \geq 1$$

So, $g'(a^-) \neq g'(a^+)$

Hence g is not differentiable at $x = a$.

Similarly, g is not differentiable at $x = b$.

Topic : Differentiability
[Rating : Doable]

2. (a, d) : As f and g are continuous, they attain their maximum on the closed and bounded interval.

If f and g achieve their maximum (common by hypothesis) at the same points, say, $x_0 \in [0, 1]$ then

$f(x_0) = g(x_0)$

Hence x_0 is the point c we are looking for and hence $f(c) = g(c)$. So choice (a) and (d) follows.

Next consider the case when f and g attain their maximum at points α and β such that $0 \leq \alpha < \beta \leq 1$.

Define $h(x) = f(x) - g(x)$

$h(\alpha) = f(\alpha) - g(\alpha) > 0$

$h(\beta) = f(\beta) - g(\beta) < 0$

Hence $\exists\, c \in [0, 1]$ such that $h(c) = 0$, as h is continuous. Thus, $f(c) = g(c)$.

And the same conclusion follows.

Topic : Rolle's Theorem
[Rating : Difficult]

3. (c, d) : Let $M = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$

If $\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} q \\ r \end{bmatrix}$ then $p = q = r$

$\det M = pr - q^2 = q^2 - q^2 = 0$

Hence M is not invertible. So choice (a) doesn't hold.

If $[q\ r] = [p\ q]$ then $p = q = r$

Again, $\det M = 0$. So choice (b) doesn't hold.

$$M = \begin{bmatrix} p & 0 \\ 0 & r \end{bmatrix} \text{ with } pr \neq 0$$

$\det M = pr \neq 0$. $\therefore$ M is invertible.

So choice (c) holds.

$$M = \begin{bmatrix} p & q \\ q & r \end{bmatrix} \Rightarrow \det M = pr - q^2$$

If pr is not the square of an integer then $pr - q^2 \neq 0$

$\therefore$ $\det M \neq 0$ $\therefore$ M is invertible.

So choice (d) holds.

Topic : Matrices
[Rating : Doable]

4. (a, b, c) : $\vec{a} = p(\vec{x} \times (\vec{y} \times \vec{z}))$, where $p \in R - \{0\}$

$= p\{(\vec{x}\cdot\vec{z})\vec{y} - (\vec{x}\cdot\vec{y})\vec{z}\}$

$= p\{\vec{y} - \vec{z}\}$ As $\vec{x}\cdot\vec{z} = \vec{x}\cdot\vec{y} = (\sqrt{2}\cdot\sqrt{2})\frac{1}{2} = 1$

$\left(\because \vec{x}\cdot\vec{y} = |x||y|\cos\frac{\pi}{3} = \sqrt{2}.\sqrt{2}\left(\frac{1}{2}\right) = 1 = \vec{y}\cdot\vec{z} = \vec{z}\cdot\vec{x}\right)$

Now, $\vec{a}\cdot\vec{y} = p\{\vec{y}\cdot\vec{y} - \vec{y}\cdot\vec{z}\} = p\{2-1\} = p$

$\therefore$ $p = \vec{a}\cdot\vec{y}$ $\quad (\because \vec{y}\cdot\vec{y} = \vec{y}^2 = (\sqrt{2})^2 = 2)$

Thus, $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{y} - \vec{z})$ ..... (i)

Similarly, $\vec{b} = q(\vec{y} \times (\vec{z} \times \vec{x})) = q\{(\vec{y}\cdot\vec{x})\vec{z} - (\vec{y}\cdot\vec{z})\vec{x}\}$

$= q(\vec{z} - \vec{x})$

Now, $\vec{b}\cdot\vec{z} = q(\vec{z}\cdot\vec{z} - \vec{z}\cdot\vec{x}) = q = (\vec{z}^2 - \vec{z}\cdot\vec{x}) = q$

Thus $\vec{b} = (\vec{b}\cdot\vec{z})(\vec{z} - \vec{x})$ .....(ii)

Now, $\vec{a}\cdot\vec{b} = (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})\{(\vec{y} - \vec{z})\cdot(\vec{z} - \vec{x})\}$ (from (i) & (ii))

$= (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})(\vec{y}\cdot\vec{z} - \vec{y}\cdot\vec{x} - \vec{z}\cdot\vec{z} + \vec{z}\cdot\vec{x})$

$= (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})(1 - 1 - 2 + 1) = -(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$

Topic : Vectors
[Rating : Doable]

5. (c): Line L_1 is : $\frac{x}{1}=\frac{y}{1}=\frac{z-1}{0}$

Line L_2 is : $\frac{x}{1}=\frac{y}{-1}=\frac{z+1}{0}$

Take a point $Q(a, a, 1)$ on L_1 and $R(b, -b, -1)$ on L_2.

$PQ \perp L_1 \Rightarrow (\lambda - a)\cdot 1 + (\lambda - a)\cdot 1 + (\lambda - 1)\cdot 0 = 0$

$\Rightarrow \lambda = a$

$PR \perp L_2 \Rightarrow (\lambda - b)\cdot 1 + (\lambda + b)\cdot(-1) + (\lambda + 1)\cdot 0 = 0$

$\Rightarrow b = 0$

Thus, $Q \equiv (\lambda, \lambda, 1)$ and $R \equiv (0, 0, -1)$

$\overrightarrow{PR}\cdot\overrightarrow{PQ} = 0 \Rightarrow 0\cdot\lambda + 0\cdot\lambda + (\lambda - 1)(\lambda + 1) = 0$

$\Rightarrow \lambda^2 - 1 = 0. \quad \therefore \lambda = 1, -1$

But for $\lambda = 1 = a$, P and Q coincide. Hence $\lambda = -1$.

Topic : Lines in 3D
[Rating : Doable]

Remark : Many students chose options (b) and (c), being careless about the points P and Q coinciding.

6. (a, b) : $M^2 = N^4 \Rightarrow (M - N^2)(M + N^2) = 0$

as M and N commute.

$M - N^2 \neq 0$ so $\det(M + N^2) = 0$

Recall that if $AB = O$ and $A \neq 0$

then $\det B = 0$ (or $MN = 0$)

$\det(M^2 + MN^2) = \det(M(M + N^2))$

$= (\det M)(\det (M + N^2))$

$= (\det M)\cdot 0 \equiv 0$

Recall that if $\det A = 0$, then $\exists\ X \neq 0$ such that $AX = 0$

Let $M^2 + MN^2$ play the role of A and X that of U, we get

$(M^2 + MN^2)U = O$ for some 3×3 non-zero matrix U.

Topic : Matrices
[Rating : Difficult]

7. (a, c, d) : $f(x) = \int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$

$$f'(x) = \frac{e^{-\left(x+\frac{1}{x}\right)}}{x}\cdot 1 - \frac{e^{-\left(\frac{1}{x}+x\right)}}{1/x}\cdot\left(-\frac{1}{x^2}\right)$$

$$= \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x} > 0$$

Thus f is monotonically increasing on $[1, \infty)$.

Again, $f\left(\frac{1}{x}\right) = \int_{x}^{1/x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$

$$= -\int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t} = -f(x)$$

$\therefore\ f(x) + f\left(\frac{1}{x}\right) = 0\ \forall\ x \in (0, \infty)$

Setting 2^x in place of x,

$f(2^x) + f(2^{-x}) = 0\ \forall\ x \in R$

$\Rightarrow g(x) + g(-x) = 0$, where $g(x) = f(2^x)$

Here g is an odd function on R.

Topic : Monotonicity
[Rating : Difficult]

8. (a, b, c) : $f(-x) = (\ln(\sec x - \tan x))^3$

$f(x) = (\ln(\sec x + \tan x))^3$

$= (-\ln(\sec x - \tan x))^3 = -(\ln(\sec x - \tan x))^3$

$= -f(-x)$

$\therefore$ f is odd.

$f(x) = (\ln(\sec x + \tan x))^3$

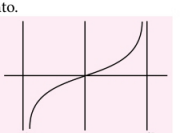
Then

$$f'(x) = \frac{3(\ln(\sec x + \tan x))^2(\sec x \tan x + \sec^2 x)}{\sec x + \tan x} > 0$$

Hence f is increasing function

As $\lim_{x\to -\pi/2} f(x) \to -\infty$ and $\lim_{x\to \pi/2} f(x) \to \infty$

Hence f is onto.



Remarks : The onto case could be argued by composite function also.

Let $g(x) = \sec x + \tan x\ \ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Then $g'(x) = \sec x \tan x + \sec^2 x = \sec x(\sec x + \tan x)$

$$= \sec x\left(\frac{1 + \sin x}{\cos x}\right) > 0\ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Thus g is increasing function. $\therefore$ g is one-one.

So, $\ln(g(x))$ is one-one and so $(\ln(g(x)))^3$ is one-one.

As $g(x) \in (0, \infty)\ \ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Then ln $g(x) \in R$ and hence f is onto.

Topic : Monotonicity, Types of function
[Rating : Difficult]

9. (b, c): I^st solution :

The circles $x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 1 = 0$ has the radical axis $-2x - 14 = 0$. $\therefore \quad x + 7 = 0$

Hence any circle orthogonal to them has its centre as $(-7, -\beta)$

Let its equation be

$S : x^2 + y^2 + 14x + 2\beta y + \lambda = 0$

Orthogonality with 2nd circle gives

$\lambda - 1 = 0. \quad \therefore \quad \lambda = 1$

Again, (0, 1) lies on $S \Rightarrow 1 + 2\beta + 1 = 0. \ \therefore \ \beta = -1$

Equation is $x^2 + y^2 + 14x - 2y + 1 = 0$

Radius $= \sqrt{7^2 + 1^2 - 1} = 7$

2nd solution :

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be orthogonal to

$x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 1 = 0$

Orthogonality condition requires

$-2g = c - 15$ and $c - 1 = 0$

Also (0, 1) lies on circle $\Rightarrow 1 + 2f + c = 0$

Solving we get $c = 1, g = 7, f = -1$

Hence radius $= \sqrt{g^2 + f^2 - c} = 7$, as before.

Topic : Circles
[Rating : Easy]

10. (b, d) : $f(x) = x^5 - 5x + a$

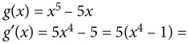
$x^5 - 5x + a = 0 \ \Rightarrow \ x^5 - 5x = -a = b$(say)

$g(x) = x^5 - 5x$

$g'(x) = 5x^4 - 5 = 5(x^4 - 1) = 5(x^2 + 1)(x - 1)(x + 1)$

$g'(1) = g'(-1) = 0$

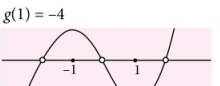
As



sign of $g'(x)$

Hence g has a maximum at $x = -1$ and a minimum at $x = 1$.

$g(-1) = 4, g(1) = -4$



Also, $x \to -\infty, g(x) \to -\infty$ and $x \to \infty, g(x) \to \infty$

For three real roots the line $x = b$ should meet the curve at three points.

$\Rightarrow \quad -4 < b < 4 \ \Rightarrow \ -4 < -a < 4 \quad \therefore \quad -4 < a < 4$

For one real root, $b < -4$ or $b > 4$

i.e. $-a < -4$ or $-a > 4$

i.e. $a > 4$ or $a < -4$

Topic : Maxima and Minima
[Rating : Doable]

11. (8) : $(y - x^5)^2 = x(1 + x^2)^2$

Differentiating with respect to x, we get

$2(y - x^5)(y_1 - 5x^4) = (1 + x^2)^2 + x \cdot 2(1 + x^2) \cdot 2x$

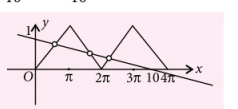
At the point (1, 3)

$2(3 - 1)(y_1 - 5) = 4 + 8 = 12$

$\therefore \quad y_1 = \dfrac{12}{4} + 5 = 8$

Topic : Tangents and Normals
[Rating : Easy]

12. (3) : The graph of $y = \cos^{-1}(\cos x)$ and $y = \dfrac{10 - x}{10} = 1 - \dfrac{x}{10}$ is shown.



The number of solutions is the number of points of intersection *i.e.* 3

Topic : Inverse Trigonometric Functions
[Rating : Easy]

13. (0) : $\displaystyle\lim_{x \to 1} \left(\frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right)^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$

$\Rightarrow \displaystyle\lim_{x \to 1} \left\{ \frac{\dfrac{\sin(x-1)}{x-1} - a}{1 + \dfrac{\sin(x-1)}{x-1}} \right\}^{1+\sqrt{x}} = \frac{1}{4}$

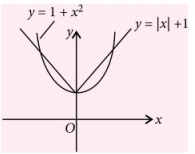
$\Rightarrow \left(\dfrac{1-a}{2} \right)^2 = \dfrac{1}{4} \ \Rightarrow \ \dfrac{1-a}{2} = \pm \dfrac{1}{2}$

$\Rightarrow \quad a = 0$ or $a = 2$

But for $a = 2$ the base approaches $-1/2$ and hence this value must be rejected.

Topic : Limits
[Rating : Doable]

14. (3) :



From the given graph the function $h(x)$ in closed form is

$$h(x) = \begin{cases} x^2+1, & -\infty < x \le -1 \\ -x+1, & -1 < x \le 0 \\ x^2+1, & 0 < x \le 1 \\ x+1, & 1 < x < \infty \end{cases}$$

Hence the points of non-differentiability are –1, 0 and 1.

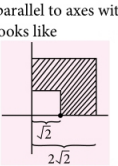
Topic : Differentiability
[Rating : Easy]

15. (6) : Let $P(x, y)$ be a point in the 1st quadrant.
$2 \le d_1(P) + d_2(P) \le 4$ gives

$$2 \le \frac{|x-y|}{\sqrt{2}} + \frac{|x+y|}{\sqrt{2}} \le 4$$

$$\Rightarrow \quad 2\sqrt{2} \le |x-y| + |x+y| \le 4\sqrt{2}$$

Now, $|x - y| + |x + y| = \lambda$ is a square with centre at origin and sides parallel to axes with length = λ.
Thus the region looks like



Area = $(2\sqrt{2})^2 - (\sqrt{2})^2 = 6$

Topic : Area
[Rating : Doable]

16. (7) : I st solution
As $n_1 \ge 1$, $n_2 \ge 2$ etc.
We have
$a + b + c + d + e = 20 - (1 + 2 + 3 + 4 + 5) = 5$
i.e. $a + b + c + d + e = 5$
Now $a \le b \le c \le d \le e$. So the cases can be listed giving
(0, 0, 0, 0, 5) ; (0, 0, 0, 1, 4); (0, 0, 0, 2, 3) ; (0, 0, 1, 1, 3); (0, 0, 1, 2, 2) ; (0, 1, 1, 1, 2); (1, 1, 1, 1, 1)
So there are 7 distinct arrangements.
2nd solution : The largest e is 10. (given)

a	b	c	d	e
1	2	3	4	10

and least e is 6.

2, 3, 4, 5, 6

So the number of ways

$$= {}^4C_0 + \frac{{}^4C_1}{4} + \frac{{}^4C_2}{3} + \frac{{}^4C_3}{2} + \frac{{}^4C_4}{1}$$

$$= 1 + 1 + 2 + 2 + 1 = 7$$

Topic : Permutations and Combinations
[Rating : Doable]

17. (2) : I st solution : Using integration by parts

$$\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2}(1-x^2)^5 \right\} dx$$

$$= 4x^3 \frac{d}{dx}(1-x^2)^5 \Big|_0^1 - \int_0^1 12x^2 \frac{d}{dx}(1-x^2)^5 dx$$

$$= 4x^3 \cdot 5(1-x^2)^4(-2x)\Big|_0^1 - 12\left[x^2(1-x^2)^5 \Big|_0^1 - \int_0^1 2x(1-x^2)^5 dx \right]$$

$$= 0 - 12\left[0 + \frac{(1-x^2)^6}{6} \Big|_0^1 \right] = -12\left(-\frac{1}{6}\right) = 2$$

2nd solution : Let $f(x) = (1 - x^2)^5$
So that $f(1) = 0$. Also $f'(1) = 0$ as 1 is a repeated root.
Now using Leibnitz generalised rule of IBP we have

$$\int_0^1 4x^3 f''(x)dx$$

$$= 4\left\{ (x^3)(f'(x)) - (3x^2)(f(x)) \Big|_0^1 + 6\int_0^1 xf(x)dx \right\}$$

$$= 24\int_0^1 x(1-x^2)^5 dx = -12\int_0^1 (1-x^2)^5 d(1-x^2)$$

$$= -12\frac{(1-x^2)^6}{6}\Big|_0^1 = -12\left(-\frac{1}{6}\right) = 2$$

Topic : Integration by Parts
[Rating : Doable]

18. (4) : As $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$

Taking dot product of vectors with $\vec{a}$, $\vec{b}$ and $\vec{c}$

$$[\vec{a}\ \vec{b}\ \vec{c}]=pa^2+q(\vec{b}\cdot\vec{a})+r(\vec{c}\cdot\vec{a})\ =p+\frac{q}{2}+\frac{r}{2};$$

$$0=\frac{p}{2}+q+\frac{r}{2}\ \text{ and }\ [\vec{a}\ \vec{b}\ \vec{c}]=\frac{p}{2}+\frac{q}{2}+r$$

Also,

$$[\vec{a}\ \vec{b}\ \vec{c}]^2=\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}=\begin{vmatrix}1 & 1/2 & 1/2\\ 1/2 & 1 & 1/2\\ 1/2 & 1/2 & 1\end{vmatrix}=\frac{1}{2}$$

$$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=\frac{1}{\sqrt{2}}$$

Thus, $\quad p+\frac{q}{2}+\frac{r}{2}=\frac{1}{\sqrt{2}}\ ;\quad \frac{p}{2}+q+\frac{r}{2}=0$

$$\frac{p}{2}+\frac{q}{2}+r=\frac{1}{\sqrt{2}}$$

On solving, we get $p=-q=r$

Hence, $\dfrac{p^2+2p^2+r^2}{q^2}=\dfrac{p^2+2p^2+p^2}{p^2}=4$

Topic : Triple Product of Vectors
[Rating : Doable]

19. (4) : 1st solution : $\dfrac{b}{a}=\dfrac{c}{b}=$ integer (say)

$$\frac{a+b+c}{3}=b+2 \qquad \text{(Given)}$$

$$\Rightarrow\ a+b+c=3b+6\ \Rightarrow\ a-2b+c=6$$

$$\Rightarrow\ a-2b+\frac{b^2}{a}=6\ \Rightarrow\ \frac{b^2}{a^2}-\frac{2b}{a}+1=\frac{6}{a}$$

$$\therefore\ \left(\frac{b}{a}-1\right)^2=\frac{6}{a}=\text{perfect square.}$$

$\therefore\ a=6$ only

Thus, $\dfrac{a^2+a-14}{a+1}=\dfrac{36+6-14}{7}=\dfrac{28}{7}=4$

2nd solution : Let a, b, c be a, ar, ar^2 where $r\in N$

Now, $a+ar+ar^2=3ar+6$

$\Rightarrow\ ar^2-2ar+a=6\ \Rightarrow\ a(r-1)^2=6$

$\Rightarrow\ (r-1)^2=6/a.\quad \therefore\ a=6$ only.

Topic : Sequences and Series
[Rating : Doable]

20. (5) : The number of red lines $= {}^nC_2-n$

The number of blue lines $= n$

Now ${}^nC_2-n=n\ \Rightarrow\ {}^nC_2=2n$

$$\Rightarrow\ \frac{n(n-1)}{2}=2n\ \Rightarrow\ n-1=4$$

$\therefore\ n=5$

Topic : Combinations
[Rating : Easy]

PAPER-2

1. (a) : A B C D E - Direction of queue

We have to choose the position for the girls.

Either a girl is at A or at B and one girl is at E.

The 2nd girl can have ${}^3C_1$ positions or ${}^2C_1$ positions accordingly and two girls, without any restriction, could choose a place in ${}^5C_2$ ways.

Thus the required probability $=\dfrac{{}^3C_1+{}^2C_1}{{}^5C_2}=\dfrac{1}{2}$

Topic : Probability
[Rating : Difficult]

2. (b) : Let two sides of the triangle be a and b

$\therefore\ ab=y$

$x^2-c^2=y\ \Rightarrow\ (a+b)^2-c^2=ab$

$\Rightarrow\ a^2+b^2+ab=c^2$

Identifying with $a^2+b^2-2ab\cos\theta=c^2$ (as c is third side)

we have $\theta=120^\circ$

$\therefore\ \angle C=2\pi/3$

$$\frac{r}{R}=\frac{\frac{\Delta}{s}}{\frac{abc}{4\Delta}}=\frac{4\Delta^2}{abcs}=\frac{(2\Delta)^2}{abc\cdot\left(\frac{a+b+c}{2}\right)}$$

$$=\frac{\left(ab\cdot\frac{\sqrt{3}}{2}\right)^2}{abc\cdot\frac{(x+c)}{2}}=\frac{3y}{2c(x+c)}$$

Topic : Properties of Triangle
[Rating : Easy]

3. (c): 1st solution : Using the principle of inclusion and exclusion, we have the number of ways in which card number 1 be placed in envelope no. 2

$$=5!-\{{}^4C_1\cdot4!-{}^4C_2\cdot3!+{}^4C_3\cdot2!-{}^4C_4\cdot1!\}$$

$$=120-\{96-36+8-1\}$$

$= 120 - \{60 + 8 - 1\} = 53$

2nd solution :

Fact : $D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \ldots + (-1)^n \frac{1}{n!}\right]$

$\therefore \quad D_5 = 5!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right] = 44$

$D_4 = 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 9$

The number of ways $= D_5 + D_4$

$= 44 + 9 = 53$

where D_n is the number of dearrangements of n objects.

Topic : Permutations and Combinations
[Rating : Difficult]

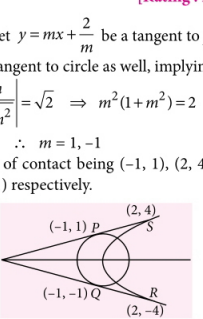
4. (d) : Let $y = mx + \frac{2}{m}$ be a tangent to $y^2 = 8x$

Then it is tangent to circle as well, implying

$\left|\frac{2/m}{\sqrt{1+m^2}}\right| = \sqrt{2} \quad \Rightarrow \quad m^2(1+m^2) = 2$

$\Rightarrow \quad m^2 = 1 \quad \therefore \quad m = 1, -1$

The points of contact being $(-1, 1)$, $(2, 4)$, $(2, -4)$ and $(-1, -1)$ respectively.



The area of quadrilateral $PQRS$

$= \left[\frac{1}{2}(1+4) \times 3\right] \times 2 = 15$

Topic : Conic Section
[Rating : Doable]

5. (d) : Take $p(x)$ as $p(x) = ax^2 + b$ with $ab > 0$

$p(p(x)) = a(ax^2 + b)^2 + b$

Let $\alpha \in R$ then $p(p(\alpha)) = a(a\alpha^2 + b)^2 + b$

$= a\left\{(a\alpha^2 + b)^2 + \frac{b}{a}\right\}$

$= a\{\text{a positive number}\} \neq 0$ (as $a \neq 0$)

Similarly, for $0 \neq \beta \in R$, we have

$p(p(i\beta)) = a(-a\beta^2 + b)^2 + b$

$= a\left\{(-a\beta^2 + b)^2 + \frac{b}{a}\right\}$

$= a\{\text{a positive number} \neq 0\}$

Thus $p(p(x)) = 0$ has neither real nor purely imaginary roots.

Topic : Quadratic Equations
[Rating : Difficult]

6. (a) : Let $e^u + e^{-u} = 2\text{cosec}x$

At $x = \pi/4$, $e^u + e^{-u} = 2\sqrt{2}$ giving $e^u = \sqrt{2} + 1$

$\therefore \quad u = \ln(1 + \sqrt{2})$

At $x = \pi/2$, $e^u + e^{-u} = 2 \quad \Rightarrow \quad e^u = 1 \quad \therefore \quad u = 0$

Now, $e^u + e^{-u} = 2\text{cosec}x$

$(e^u - e^{-u})du = -2(\text{cosec}x)\cot x\, dx$

$\therefore \quad (e^u + e^{-u})^2 = 4\text{cosec}^2 x = 4(1 + \cot^2 x)$

$\therefore \quad (e^u - e^{-u})^2 = 4\cot^2 x \Rightarrow 2\cot x = e^u - e^{-u}$

$\Rightarrow \quad (e^u - e^{-u})du = -(e^u + e^{-u}) \cdot (e^u - e^{-u})dx$

$\therefore \quad du = -(e^u + e^{-u})\, dx$

$$\int_{\pi/4}^{\pi/2} (2\text{cosec}x)^{17}\, dx = \int_{\ln(1+\sqrt{2})}^{0} -(e^u + e^{-u})^{17} \cdot \left(\frac{2}{e^u + e^{-u}}\right) du$$

$$= \int_{0}^{\ln(1+\sqrt{2})} 2(e^u + e^{-u})^{16}\, du$$

Topic : Integration
[Rating : Difficult]

7. (b) : $\frac{dy}{dx} + \frac{x}{x^2 - 1} y = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$

The I.F. of this differential equation is

$e^{\int \frac{x}{x^2-1} dx} = e^{-\int \frac{x}{1-x^2} dx} = e^{\frac{1}{2}\ln(1-x^2)} = \sqrt{1 - x^2}$

The solution is given by

$$y\sqrt{1 - x^2} = \int \frac{x(x^3 + 2)}{\sqrt{1 - x^2}} \sqrt{1 - x^2}\, dx + \lambda$$

$$= \int (x^4 + 2x)dx + \lambda = \frac{x^5}{5} + x^2 + \lambda$$

As $y(0) = 0 \quad \Rightarrow \quad \lambda = 0$

$$\Rightarrow \quad y\sqrt{1 - x^2} = \frac{x^5}{5} + x^2$$

$$\int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{\frac{x^5}{5} + x^2}{\sqrt{1 - x^2}} dx = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^2}{\sqrt{1 - x^2}} dx$$

(The other part is odd)

$$= 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx$$

Let $x = \sin\theta$, we get

$$I = 2\int_0^{\pi/3} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta = 2\int_0^{\pi/3} \sin^2\theta d\theta$$

$$= \int_0^{\pi/3} (1-\cos 2\theta)d\theta$$

$$= \theta - \frac{\sin 2\theta}{2}\Big|_0^{\pi/3} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

Topic : Differential Equations
[Rating : Easy]

8. (b) : Given $F(0) = 0$

$F'(x) = 2x f(x) = f'(x)$

On integrating, $f(x) = \lambda e^{x^2}$

As $f(0) = 1$ we have $\lambda = 1$

$\therefore \quad f(x) = e^{x^2}$

$$F(x) = \int_0^{x^2} e^t dt \quad \Rightarrow F(2) = \int_0^4 e^t dt = e^t|_0^4 = e^4 - 1$$

Topic : Integration
[Rating : Easy]

9. (c): $(1+x^m)^n = \sum_{r=0}^{n} {}^nC_r (x^m)^r = \sum_{r=0}^{n} {}^nC_r x^{mr}$

We have $2\alpha + 3\beta + 4\gamma = 11$, where α, β, γ come from expansions correspondingly to $(1 + x^2)^4$, $(1 + x^3)^7$ and $(1 + x^4)^{12}$.

The possibilities are (0, 1, 2), (1, 3, 0), (2, 1, 1) and (4, 1, 0).

And they can be realized in

$({}^4C_0 \cdot {}^7C_1 \cdot {}^{12}C_2) + ({}^4C_1 \cdot {}^7C_3 \cdot {}^{12}C_0)$
$+ ({}^4C_2 \cdot {}^7C_3 \cdot {}^{12}C_1) + ({}^4C_4 \cdot {}^7C_1 \cdot 1)$

$= (1 \cdot 7 \cdot 66) + (4 \cdot 35 \cdot 1) + (6 \cdot 7 \cdot 12) + (1 \cdot 7)$

$= 462 + 140 + 504 + 7 = 1113$

Topic : Binomial Theorem
[Rating : Difficult]

10. (d) : $\sin x + 2\sin 2x - \sin 3x = 3$

$\Rightarrow \sin x + 4\sin x\cos x - 3\sin x + 4\sin^3 x = 3$

$\Rightarrow \sin x\{-2 + 4\cos x + 4\sin^2 x\} = 3$

$\Rightarrow \sin x(-2 + 4\cos x + 4 - 4\cos^2 x) = 3$

$\Rightarrow \sin x\{2 - (4\cos^2 x - 4\cos x + 1) + 1\} = 3$

$\Rightarrow \sin x\{3 - (1 - 2\cos x)^2\} = 3$

$\Rightarrow \sin x = 1, 2\cos x - 1 = 0$

$\Rightarrow \quad x = \frac{\pi}{3}$ and $x = \frac{\pi}{2}$

So, no common solution

$\therefore$ There is no value of x which satisfies the equation.

Topic : Trigonometric Equations
[Rating : Difficult]

11. (b) : Given sets $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{1, 2, 3, 4, 5, 6, 7\}$

Total number of ways (one number from each set A, B, C) $= {}^3C_1 \times {}^5C_1 \times {}^7C_1 = 105$

Favourable cases : (1) All three numbers be odd one from each set A, B, $C = {}^2C_1 \times {}^3C_1 \times {}^4C_1 = 24$

(2) One odd number from set A & 1 even number from B & C each

$\therefore$ Required ways $= {}^2C_1 \times {}^2C_1 \times {}^3C_1 = 12$

(3) One odd number from set B & 1 even number from A & C each

$\therefore$ Required ways $= {}^3C_1 \times {}^1C_1 \times {}^3C_1 = 9$

(4) One odd number from set C & 1 even number from A & B each

$\therefore$ Required ways $= {}^4C_1 \times {}^1C_1 \times {}^2C_1 = 8$

$\therefore$ Favourable cases $= 24 + (12 + 9 + 8) = 53$

$\therefore$ Required probability $= \frac{53}{105}$

Topic : Probability
[Rating : Doable]

12. (c): As $2x_2 = x_1 + x_3$

Number of ways $= {}^2C_1 \cdot {}^4C_2 + {}^1C_1 \cdot {}^3C_1 = 11$

Topic : Probability
[Rating : Doable]

13. (d) : The slope of PK = the slope of QR

$$\Rightarrow \frac{-\frac{2a}{t} - 2ar}{\frac{a}{t^2} - ar^2} = \frac{2at - 0}{at^2 - 2a} \quad \left(\because m = \frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\Rightarrow \frac{t}{t^2-2}=-\frac{\frac{1}{t}+r}{\left(\frac{1}{t^2}-r^2\right)}=-\frac{1}{\frac{1}{t}-r}$$

$\Rightarrow 1 - tr = 2 - t^2 \Rightarrow tr = t^2 - 1$

$\therefore r=\frac{t^2-1}{t}$

Topic : Conics
[Rating : Doable]

14. (b) : The tangent at P to the parabola is

$$y=\frac{1}{t}x+at \quad \text{..... (1)}$$

Normal at S : $y+\frac{x}{t}=\frac{2a}{t}+\frac{a}{t^3}$ ..... (2)

On solving, we get

$$2y=at+\frac{2a}{t}+\frac{a}{t^3}=a\left(t+\frac{2}{t}+\frac{1}{t^3}\right)=\frac{a(t^4+2t^2+1)}{t^3}$$

$\therefore y=\frac{a(t^2+1)^2}{2t^3}$

Topic : Conics
[Rating : Doable]

15. (a) : $g\left(\frac{1}{2}\right)=\lim\limits_{h\to 0^+}\int\limits_h^{1-h} t^{-1/2}(1-t)^{-1/2}dt$

$$=\int_0^1\frac{dt}{\sqrt{t-t^2}}=\int_0^1\frac{dt}{\sqrt{\frac{1}{4}-\left(t-\frac{1}{2}\right)^2}}=\sin^{-1}\left\{\frac{\left(t-\frac{1}{2}\right)}{\frac{1}{2}}\right\}\Bigg|_0^1$$

$= \sin^{-1}1 - \sin^{-1}(-1) = \pi$

Topic : Definite Integrals
[Rating : Doable]

16. (d): We get $g(a) = g(1 - a)$ and g is differentiable.

Here $g'\left(\frac{1}{2}\right)=0$

Topic : Differentiability
[Rating : Easy]

17. (d) : P : $f(x)=ax^2+bx$, also $\int\limits_0^1 f(x)dx=1$

which gives $2a + 3b = 6$

$\Rightarrow$ $(a, b) = (0, 2)$ and $(3, 0)$

Q : $f(x)=\sqrt{2}\cos\left(x^2-\frac{\pi}{4}\right)$

For the greatest value, $x^2=2k\pi+\frac{\pi}{4}$

As $x\in(-\sqrt{13},\sqrt{13})$, we have

$$x=\pm\sqrt{\frac{\pi}{4}} \quad \text{and} \quad \pm\sqrt{\frac{9\pi}{4}}$$

Number of values of x are 4.

R : Note that $\frac{1}{1+e^x}+\frac{1}{1+e^{-x}}=1$

Thus, $\int\limits_0^2\left(\frac{3x^2}{1+e^{-x}}+\frac{3x^2}{1+e^x}\right)dx=\int\limits_0^2 3x^2dx=8$

S : As the integral is an odd function,

$$\int\limits_{-1/2}^{1/2}\cos 2x\ln\left(\frac{1+x}{1-x}\right)=0$$

Topic : Integration
[Rating : Easy]

18. (a) : P : $y = \cos(3\cos^{-1}x)$

$$\Rightarrow \frac{dy}{dx}=y'=\frac{3\sin(3\cos^{-1}x)}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,y'=3\sin(3\cos^{-1}x)$$

Again, differentiate to obtain

$$-\frac{x}{\sqrt{1-x^2}}y'+\sqrt{1-x^2}\,y''=3\cos(3\cos^{-1}x)\cdot\left(\frac{-3}{\sqrt{1-x^2}}\right)$$

$\Rightarrow (1 - x^2)y'' - xy' = -9y$

Hence $\frac{1}{y}\left[(x^2-1)y''+xy'\right]=9$

Q : Note that $\overrightarrow{a_k}\cdot\overrightarrow{a_{k+1}}=r^2\cos\frac{2\pi}{n}$

$$\left|\overrightarrow{a_k}\times\overrightarrow{a_{k+1}}\right|=r^2\sin\frac{2\pi}{n}$$

Thus $r^2(n-1)\sin\frac{2\pi}{n}=r^2(n-1)\cos\frac{2\pi}{n}$

$\Rightarrow \tan\frac{2\pi}{n}=1 \Rightarrow \frac{2\pi}{n}=n\pi+\frac{\pi}{4}$

$\therefore n=\frac{8}{4n+1}$

The minimum of n is 8.

R : $(h, 1)$ lies on ellipse $\Rightarrow \frac{h^2}{6}+\frac{1}{3}=1$

$\Rightarrow \frac{h^2}{6}=\frac{2}{3} \Rightarrow h^2=4 \Rightarrow h=-2, 2$

For $h=2$ tangent at $(2, 1)$ is $\frac{2x}{6}+\frac{y}{3}=1$

i.e. $x+y=3$

So the normal has to be perpendicular to it.

S : $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{2x+1}+\frac{1}{4x+1}}{1-\frac{1}{2x+1}\cdot\frac{1}{4x+1}}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{3x+1}{4x^2+3x}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow 3x^2-7x-6=0 \Rightarrow 3x^2-9x+2x-6=0$

$\Rightarrow 3x(x-3)+2(x-3)=0$

$\Rightarrow (3x+2)(x-3)=0 \Rightarrow x=-2/3, 3$

Of them, only $x=3$ is the positive one.

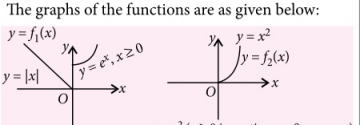
Topic : Inverse Trigonometric Functions, Conic Section, Vector Algebra
[Rating : Doable]

19. (d) : Let's rewrite the functions

$$f_2(f_1(x))=\begin{cases}x^2, & x<0\\ e^{2x}, & x\geq 0\end{cases}$$

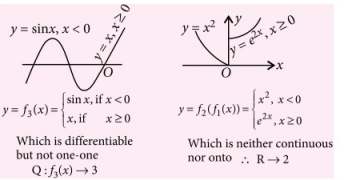
$$f_4(x)=\begin{cases}f_2(f_1(x)), & x<0\\ f_2(f_1(x))-1, & x\geq 0\end{cases}=\begin{cases}x^2, & x<0\\ e^{2x}-1, & x\geq 0\end{cases}$$

The graphs of the functions are as given below:



$y=x^2$ $(x\geq 0$ is continuous & one-one)

S : $f_2(x)\to 4$



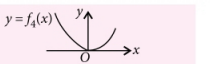
$y=f_3(x)=\begin{cases}\sin x, \text{ if } x<0\\ x, \text{ if } \quad x\geq 0\end{cases}$

Which is differentiable but not one-one

Q : $f_3(x)\to 3$

$y=f_2(f_1(x))=\begin{cases}x^2, & x<0\\ e^{2x}, & x\geq 0\end{cases}$

Which is neither continuous nor onto $\therefore$ R $\to$ 2



$f_4(x)=\begin{cases}x^2, \text{ if } x<0\\ e^{2x}-1, \text{if } x\geq 0\end{cases}$

Which is onto but not one-one

$\therefore$ P $\to$ 1

Topic : Differentiability, Functions
[Rating : Difficult]

20. (c): **P** : z_k is a tenth roots of unity

$\Rightarrow \bar{z}_k$ will also be.

Also, $z_k\cdot\bar{z}_k=1$

Q : $z_1\cdot z=z_k$

Let $z_1\neq 0$ then $z=\frac{z_k}{z_1}$ (As $z_1\neq 0$)

Then one can always find z.

R : $z^{10}-1=(z-1)(z-z_1)\ldots(z-z_9)$

$\Rightarrow \frac{z^{10}-1}{z-1}=(z-z_1)(z-z_2)\ldots(z-z_9)$

$=1+z^1+z^2+\ldots+z^9$

Set $z=1$ to get $(1-z_1)(1-z_2)\ldots(1-z_9)=10$

$\Rightarrow \frac{(1-z_1)(1-z_2)\ldots..(1-z_9)}{10}=1$

S : $1+z_1+z_2+\ldots+z_9=0$

$\Rightarrow \text{Re}|z_1|+\text{Re}|z_2|+\ldots.+\text{Re}|z_9|=-1$

$\Rightarrow \sum_{k=1}^{9}\cos\frac{2\pi k}{10}=-1$

$\therefore 1-\sum_{k=1}^{9}\cos\frac{2\pi k}{10}=2$

Topic : Complex Numbers
[Rating : Doable]

2015
CBSE-BOARD
CHAPTERWISE PRACTICE PAPER

Series-2 **Inverse Trigonometric Functions**

INVERTIBLE FUNCTION

A function $f : A \to B$ is a invertible iff it is a bijection.

INVERSE OF A FUNCTION

If $f : X \to Y$ such that $f(x) = y$ is one-one and onto then we can define a unique function $g : Y \to X$ such that $g(y) = x$ where $x \in X$ and $y = f(x)$, $y \in Y$. The function g is called the inverse of f and is denoted by f^{-1}. Further, g is also one-one and onto and inverse of g is f^{-1}. Thus, $g^{-1} = (f^{-1})^{-1} = f$

Also, $(f^{-1}of)(x) = f^{-1}(f(x)) = f^{-1}(y) = x$

$$(fof^{-1})(y) = f(f^{-1}(y)) = f(x) = y$$

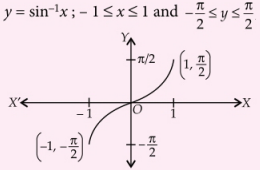
Here, domain of g = range of f
and range of g = domain of f.

TABLE FOR THE DOMAIN AND RANGE OF INVERSE TRIGONOMETRIC FUNCTIONS

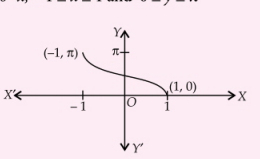
Functions	Domain	Range (principal value branch)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1}x$	$(-\infty, \infty)$	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$y = \text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \pi\right] - \left\{\frac{\pi}{2}\right\}$
$y = \cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$

GRAPHS OF INVERSE TRIGONOMETRIC FUNCTIONS

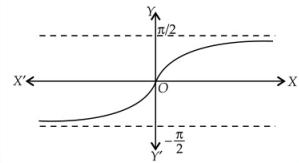
1. $y = \sin^{-1}x$; $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



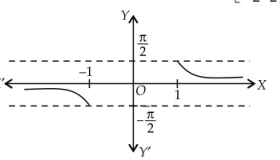
2. $y = \cos^{-1}x$; $-1 \le x \le 1$ and $0 \le y \le \pi$



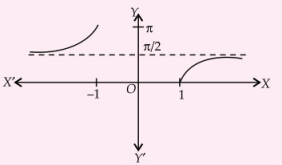
3. $y = \tan^{-1}x\,;\, x \in R$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$



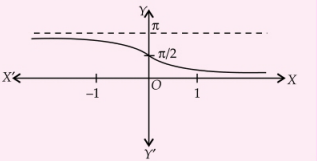
4. $y = \text{cosec}^{-1}x\,;\; x \in R - (-1, 1)$ and $y \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$



5. $y = \sec^{-1}x\,;\, x \in R - (-1, 1)$ and $y \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$



6. $y = \cot^{-1}x,\, x \in R$ and $0 < y < \pi$



PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

1.	• $\sin^{-1}(\sin\theta) = \theta,\ \forall\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	2.	• $\sin(\sin^{-1}x) = x,\ \forall\ x \in [-1, 1]$
	• $\cos^{-1}(\cos\theta) = \theta,\ \forall\ \theta \in [0, \pi]$		• $\cos(\cos^{-1}x) = x,\ \forall\ x \in [-1, 1]$
	• $\tan^{-1}(\tan\theta) = \theta,\ \forall\ \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$		• $\tan(\tan^{-1}x) = x,\ \forall\ x \in R$
	• $\text{cosec}^{-1}(\text{cosec}\ \theta) = \theta,\ \forall\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$		• $\text{cosec}(\text{cosec}^{-1}x) = x,\ \forall\ x \in R - (-1, 1)$
	• $\sec^{-1}(\sec\theta) = \theta,\ \forall\ \theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$		• $\sec(\sec^{-1}x) = x,\ \forall\ x \in R - (-1, 1)$
	• $\cot^{-1}(\cot\theta) = \theta,\ \forall\ \theta \in (0, \pi)$		• $\cot(\cot^{-1}x) = x,\ \forall\ x \in R$

3.	• $\sin^{-1}(-x) = -\sin^{-1}(x),\ \forall\ x \in [-1, 1]$	• $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x,\ \forall\ x \in (-\infty, -1] \cup [1, \infty)$
	• $\cos^{-1}(-x) = \pi - \cos^{-1}x,\ \forall\ x \in [-1, 1]$	• $\sec^{-1}(-x) = \pi - \sec^{-1}x,\ \forall\ x \in (-\infty, -1] \cup [1, \infty)$
	• $\tan^{-1}(-x) = -\tan^{-1}x,\ \forall\ x \in R$	• $\cot^{-1}(-x) = \pi - \cot^{-1}x,\ \forall\ x \in R$

4.	• $\sin^{-1}(1/x) = \text{cosec}^{-1}x,\ \forall\ x \in (-\infty, -1] \cup [1, \infty)$	5.	• $\sin^{-1}x + \cos^{-1}x = \pi/2,\ \forall\ x \in [-1, 1]$
	• $\cos^{-1}(1/x) = \sec^{-1}x,\ \forall\ x \in (-\infty, -1] \cup [1, \infty)$		• $\tan^{-1}x + \cot^{-1}x = \pi/2,\ \forall\ x \in R$
	• $\tan^{-1}(1/x) = \begin{cases} \cot^{-1}x & , \text{ for } x > 0 \\ -\pi + \cot^{-1}x & , \text{ for } x < 0 \end{cases}$		• $\sec^{-1}x + \text{cosec}^{-1}x = \pi/2,$ $\forall\ x \in (-\infty, -1] \cup [1, \infty)$

6.

- $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & , \text{ if } xy<1 \\ \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & , \text{ if } x>0,\ y>0 \text{ and } xy>1 \\ -\pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right), & \text{if } x<0,\ y<0 \text{ and } xy>1 \end{cases}$

- $\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & , \text{ if } xy>-1 \\ \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & , \text{ if } x>0,\ y<0 \text{ and } xy<-1 \\ -\pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right), & \text{if } x<0,\ y>0 \text{ and } xy<-1 \end{cases}$

7.

- $\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} & , \quad \text{if } -1\le x,\ y\le 1 \text{ and } x^2+y^2\le 1 \\ & \quad \text{or} \\ & \quad \text{if } xy<0 \text{ and } x^2+y^2>1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} & , \quad \text{if } 0<x,\ y\le 1 \text{ and } x^2+y^2>1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}, & \text{if } -1\le x,\ y<0 \text{ and } x^2+y^2>1 \end{cases}$

- $\sin^{-1}x - \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} & , \quad \text{if } -1\le x,\ y\le 1 \text{ and } x^2+y^2\le 1 \\ & \quad \text{or} \\ & \quad \text{if } xy>0 \text{ and } x^2+y^2>1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} & , \quad \text{if } 0<x\le 1,\ -1\le y\le 0 \text{ and } x^2+y^2>1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}, & \text{if } -1\le x<0,\ 0<y\le 1 \text{ and } x^2+y^2>1 \end{cases}$

8.

- $\cos^{-1}x + \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1\le x,\ y\le 1 \text{ and } x+y\ge 0 \\ 2\pi - \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}, & \text{if } -1\le x,\ y\le 1 \text{ and } x+y\le 0 \end{cases}$

- $\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\} & , \text{ if } -1\le x,\ y\le 1 \text{ and } x\le y \\ -\cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}, & \text{if } -1\le y\le 0,\ 0<x\le 1 \text{ and } x\ge y \end{cases}$

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9.

- $2\sin^{-1}x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}) & , \text{ if } -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}) & , \text{ if } \frac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin(2x\sqrt{1-x^2}) & , \text{ if } -1 \le x \le -\frac{1}{\sqrt{2}} \end{cases}$

- $3\sin^{-1}x = \begin{cases} \sin^{-1}(3x-4x^3) & , \text{ if } -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - \sin^{-1}(3x-4x^3) & , \text{ if } \frac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x-4x^3) & , \text{ if } -1 \le x < -\frac{1}{2} \end{cases}$

10.

- $2\cos^{-1}x = \begin{cases} \cos^{-1}(2x^2-1) & , \text{ if } 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2-1) & , \text{ if } -1 \le x \le 0 \end{cases}$

- $3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3-3x) & , \text{ if } \frac{1}{2} \le x \le 1 \\ 2\pi - \cos^{-1}(4x^3-3x) & , \text{ if } -\frac{1}{2} \le x \le \frac{1}{2} \\ 2\pi + \cos^{-1}(4x^3-3x) & , \text{ if } -1 \le x \le -\frac{1}{2} \end{cases}$

11.

- $2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right) & , \text{ if } -1 < x < 1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) & , \text{ if } x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) & , \text{ if } x < -1 \end{cases}$

- $3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & , \text{ if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & , \text{ if } x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & , \text{ if } x < -\frac{1}{\sqrt{3}} \end{cases}$

Contd. on page no. 86

concept BOOSTERS

Class XI

Sequences and Series

* ALOK KUMAR, B.Tech, IIT Kanpur

This column is aimed at Class XI students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

DEFINITION

A sequence is a set of terms in a definite order with a rule for obtaining the terms.

e.g. 1, 1/2, 1/3, ..., 1/n, ... is a sequence.

ARITHMETIC PROGRESSION (A.P.) :

It is a sequence whose terms increase or decrease by a fixed number. This fixed number is called the common difference. If a is the first term & d is the common difference, then A.P. can be written as

$a, a + d, a + 2d, \ldots\ldots a + (n - 1)d, \ldots\ldots$

n^{th} term of this A.P., $t_n = a + (n - 1)d$, where $d = a_n - a_{n-1}$.

The sum of the first n terms of the A.P. is given by $S_n = \frac{n}{2}[2a + (n-1)d)] = \frac{n}{2}[a + l]$

where l is the last term.

REMARKS :

- The sum of the two terms of an A.P. equidistant from the beginning and end is constant and equal to the sum of first and last terms.
- Three numbers in A.P. can be taken as $a - d, a, a + d$; four numbers in A.P. can be taken as $a - 3d, a - d, a + d, a + 3d$; five numbers in A.P. are $a - 2d, a - d, a, a + d, a + 2d$ & six terms in A.P. are $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.
- If each term of an A.P. is increased, decreased, multiplied or divided by the same non-zero number, then the resulting sequence is also an A.P.
- The common difference can be zero, positive or negative.
- Any term of an A.P. (except the first) is equal to half the sum of terms which are equidistant from it.
- $t_r = S_r - S_{r-1}$
- If a, b, c are in A.P. $\Rightarrow 2b = a + c$.

GEOMETRIC PROGRESSION (G.P.) :

It is a sequence of numbers whose first term is non-zero & each of the succeeding terms is equal to the preceeding terms multiplied by a constant. Thus, in a G.P. the ratio of successive terms is constant. This constant factor is called the common ratio of the series and is obtained by dividing any term by that which immediately proceeds it. Therefore $a, ar, ar^2, ar^3, ar^4, \ldots\ldots$ is a G.P. with a as the first term & r as common ratio.

REMARKS :

- n^{th} term = ar^{n-1}
- Sum of the first n terms *i.e.* $S_n = \frac{a(r^n - 1)}{r - 1}$, if $r \neq 1$.
- Sum of an infinite G.P. when $|r| < 1$ when $n \to \infty$, $r^n \to 0$ if $|r| < 1$ therefore, $S_\infty = \frac{a}{1 - r}(|r| < 1)$.
- If each term of a G.P. be multiplied or divided by the same non-zero quantity, the resulting sequence is also a G.P.

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

- Any three consecutive terms of a G.P. can be taken as $a/r, a, ar$; any four consecutive terms of a G.P. can be taken as $a/r^3, a/r, ar, ar^3$ & so on.
- If a, b, c are in G.P. $\Rightarrow b^2 = ac$.

HARMONIC PROGRESSION (H.P.) :

A sequence is said to in H.P. if the reciprocals of its terms are in A.P.

If the sequence $a_1, a_2, a_3, \ldots, a_n$ is in H.P. then $1/a_1, 1/a_2, \ldots, 1/a_n$ is in A.P. & converse. Here we do not have the formula for the sum of the n terms of an H.P.. For H.P. whose first term is a and second term is b, the n^{th} term is

$$t_n = \frac{ab}{b + (n-1)(a-b)}$$

If a, b, c are in H.P. $\Rightarrow b = \dfrac{2ac}{a+c}$ or $\dfrac{a}{c} = \dfrac{a-b}{b-c}$

ARITHMETIC MEAN :

If three terms are in A.P. then the middle term is called the A.M. between the other two, so if a, b, c are in A.P., b is A.M. of a & c.

A.M. for any n positive numbers $a_1, a_2, \ldots, a_n$ is $A = \dfrac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$

n-ARITHMETIC MEANS BETWEEN TWO NUMBERS :

If a, b are any two given numbers & $a, A_1, A_2, \ldots, A_n, b$ are in A.P. then $A_1, A_2, \ldots, A_n$ are the n A.M.'s between a & b.

$$A_1 = a + \frac{b-a}{n+1}, \quad A_2 = a + \frac{2(b-a)}{n+1}, \ldots\ldots\ldots,$$

$$A_n = a + \frac{n(b-a)}{n+1}$$

$$= a + d, = a + 2d, \ldots\ldots\ldots, A_n = a + nd,$$

where $d = \dfrac{b-a}{n+1}$

Note : Sum of n A.M.'s inserted between a & b is equal to n times the single A.M. between a & b i.e. $\sum_{r=1}^{n} A_r = nA = n\left(\dfrac{a+b}{2}\right)$ where A is the single A.M. between a & b.

GEOMETRIC MEANS :

If a, b, c are in G.P., b is the G.M. between a & c.

$b^2 = ac$, therefore $b = \sqrt{ac}$; $a > 0, c > 0$.

n-GEOMETRIC MEANS BETWEEN A, B

If a, b are two given numbers & $a, G_1, G_2, \ldots, G_n, b$ are in G.P., then $G_1, G_2, G_3, \ldots, G_n$ are n G.M.s between a & b.

$G_1 = a(b/a)^{1/n+1}$, $G_2 = a(b/a)^{2/n+1}$, ……,

$G_n = a(b/a)^{n/n+1}$

$= ar, = ar^2, \ldots\ldots, = ar^n$, where $r = (b/a)^{1/n+1}$

Note : The product of n G.M.s between a & b is equal to the n^{th} power of the single G.M. between a & b

i.e. $\prod_{r=1}^{n} G_r = (G)^n$ where G is the single G.M. between a & b, where $G = \sqrt{ab}$

HARMONIC MEAN :

If a, b, c are in H.P., b is the H.M. between a & c, then $b = 2ac/[a + c]$.

THEOREM :

If A, G, H are respectively A.M., G.M., H.M. between a & b both being unequal and positive then,

(i) $G^2 = AH$

(ii) $A > G > H$ $(G > 0)$. Note that A, G, H constitute a G.P.

ARITHMETICO-GEOMETRIC SERIES :

A series each term of which is formed by multiplying the corresponding term of an A.P. & G.P. is called the **Arithmetico-Geometric Series.** e.g. $1 + 3x + 5x^2 + 7x^3 + \ldots\ldots$

Here 1, 3, 5, ... are in A.P. & $1, x, x^2, x^3$ ..... are in G.P.

Example :

Let $S_n = a + (a + d)r + (a + 2d)r^2 + \ldots\ldots + [a + (n-1)d]r^{n-1}$

$$= \frac{a}{1-r} + \frac{ar(1-r^{n-1})}{1-r^2} - \frac{(a+(n-1)d)r^n}{1-r}$$

SUM TO INFINITY :

If $|r| < 1$ & $n \to \infty$ then

$\underset{n \to \infty}{\text{Limit}}\ r^n = 0$. $S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$.

THEOREMS :

- $\sum_{r=1}^{n} ka_r = k \sum_{r=1}^{n} a_r$

- $\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$
- $\sum_{r=1}^{n} k = nk$; where k is a constant

RESULTS

- $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

  (sum of the first n natural numbers)
- $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

  (sum of the squares of the first n natural numbers)
- $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2$

  (sum of the cubes of the first n natural numbers)
- $\sum_{r=1}^{n} r^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$

METHOD OF DIFFERENCE :

If $T_1, T_2, T_3, \ldots, T_n$ are the terms of a sequence then sometimes the terms $T_2 - T_1, T_3 - T_2, \ldots$ constitute an A.P./G.P.. n^{th} term of the series is determined & the sum to n terms of the sequence can easily be obtained.

Remember that to find the sum of n terms of a series each term of which is composed of r factors in A.P., the first factors of several terms being in the same A.P., we "write down the n^{th} term, affix the next factor at the end, divide by the number of factors thus increased and by the common difference and add a constant. Determine the value of the constant by A.P.plying the initial conditions".

SECTION-I

Single Correct Answer Type

1. If $a_1, a_2, a_3, \ldots, a_{4001}$ are terms of an A.P. such that $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{4000}a_{4001}} = 10$ and $a_2 + a_{4000} = 50$ then $|a_1 - a_{4001}|$ is equal to

(a) 20 (b) 30 (c) 40 (d) 10

2. If x, y, z are distinct positive numbers, then $x^{\ln y - \ln z} + y^{\ln z - \ln x} + z^{\ln x - \ln y} \in$

(a) $(0, \infty)$ (b) $(1, \infty)$ (c) $(3, \infty)$ (d) $(1, 3)$

3. The square roots of two successive natural numbers greater than N^2 differ by

(a) $\frac{1}{N}$ (b) $\frac{1}{2N}$

(c) greater than $\frac{1}{2N}$ (d) less than $\frac{1}{2N}$

4. The sum $\frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \ldots + \frac{2008}{2006!+2007!+2008!}$ is equal to

(a) $\frac{1}{2} - \frac{1}{2006!}$ (b) $\frac{1}{2} - \frac{1}{2010!}$

(c) $\frac{1}{2006! - 2008!}$ (d) $\frac{1}{2007!} - \frac{1}{2008!}$

5. Consider the sequence of numbers $\alpha_0, \alpha_1, \alpha_2, \ldots\ldots\alpha_n$, where $\alpha_0 = 17.23$, $\alpha_1 = 33.23$ and $\alpha_{r+2} = \frac{\alpha_r + \alpha_{r+1}}{2}$, the value of $|\alpha_{10} - \alpha_9|$ equals

(a) $\frac{41}{50}$ (b) $\frac{28}{61}$ (c) $\frac{1}{32}$ (d) $\frac{33}{277}$

6. If $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots\ldots = \frac{\pi}{4}$ then the value of $\frac{1}{1\cdot3} + \frac{1}{5\cdot7} + \frac{1}{9\cdot11} + \ldots\ldots$ is

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{36}$

7. A sequence of numbers A_n, n = 1,2,3,.....is defined as follows : $A_1 = \frac{1}{2}$ and for each $n \geq 2, A_n = \left(\frac{2n-3}{2n}\right) A_{n-1}$ then

(a) $\sum_{k=1}^{n} A_k = 1$ for all $n \geq 1$

(b) $\sum_{k=1}^{n} A_k > 1$ for all $n \geq 1$

(c) $\sum_{k=1}^{n} A_k < 1$ for all $n \geq 1$

(d) None of these

8. Let $a_0, a_1, a_2, \ldots\ldots, a_n \ldots$ be a sequence of numbers satisfying $(3 - a_{n+1})\cdot(6 + a_n) = 18$ and $a_0 = 3$ then $\sum_{i=0}^{n} \frac{1}{a_i} =$

(a) $\frac{1}{3}(2^{n+2}+n-3)$ (b) $\frac{1}{3}(2^{n+2}-n+3)$

(c) $\frac{1}{3}(2^{n+2}-n-3)$ (d) $\frac{1}{3}(2^{n+2}+n+3)$

9. If $a_i > 0$ for $1 \le i \le 5$ and $a_1a_2.....a_5 = 6$, then the least value of $a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5$ is
(a) $6(5!)^{1/5}$ (b) $15(6!)^{1/5}$
(c) $(6!)^{1/5}$ (d) $5(6!)^{1/5}$

10. If a, b, c are each positive, and $a + b + c = 6$ then the minimum value of

$\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2$ is

(a) $\frac{75}{2}$ (b) $\frac{75}{4}$ (c) $\frac{65}{4}$ (d) $\frac{65}{2}$

11. The sixth term of an A.P is 2. The value of the common difference of A.P which makes the product $a_1.a_4.a_5$ greatest is
(a) 8/5 (b) 3 (c) 2 (d) 4/5

12. Sum of the series $\frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+.....$ to ∞ is

(a) $\frac{38}{81}$ (b) $\frac{4}{19}$ (c) $\frac{36}{371}$ (d) $\frac{18}{19}$

13. If sum of n terms of A.P. is $cn(n-1)$ when $c \neq 0$.The sum of the squares of these terms is
(a) $c^2n^2(n+1)^2$

(b) $\frac{2}{3}c^2n(n-1)(2n-1)$

(c) $\frac{2c^2}{3}n(n+1)(2n+1)$

(d) $\frac{2c^2(n-1)(n-2)(2n-3)}{3}$

14. Largest term in the expression of $\frac{1}{201},\frac{4}{208},\frac{9}{227},\frac{16}{264},.....$
(a) 6^{th} term (b) 7^{th} term
(c) 8^{th} term (d) 10^{th} term

15. If $a + b + c + d + e = 8$ and $a^2 + b^2 + c^2 + d^2 + e^2 = 16$ where a,b,c,d,e are non-negative real numbers and range of e is $\left[l,\frac{m}{n}\right]$ (where G.C.D of $m,n = 1$) then $l + m + n$ is equal to
(a) 21 (b) 26 (c) 20 (d) 31

16. Given that 'n' arithmetic means are inserted between two sets of numbers a, $2b$ and $2a$, b where $a, b \in R$. Suppose that the m^{th} mean of the two sets of numbers is same, then $a : b =$

(a) $\frac{n}{n-m+1}$ (b) $\frac{m}{n-m+1}$

(c) $\frac{n-m+1}{n}$ (d) $\frac{n-m+1}{m}$

17. If A, G, H be respectively the A.M., G.M. and the H.M. between two positive numbers if $xA = yG = zH$ where x, y, z are non-zero positive quantities then x, y, z are in
(a) A.P. (b) G.P. (c) H.P. (d) A.G.P.

SECTION-II

More than One Correct Answer Type

18. Let $S_n(n \ge 1)$ be a sequence of sets defined by $S_1 = \{0\}$, $S_2 = \left\{\frac{3}{2},\frac{5}{2}\right\}$, $S_3 = \left\{\frac{8}{3},\frac{11}{3},\frac{14}{3}\right\}$,

$S_4 = \left\{\frac{15}{4},\frac{19}{4},\frac{23}{4},\frac{27}{4}\right\}$, ... then

(a) third element in S_{20} is $\frac{439}{20}$

(b) third element in S_{20} is $\frac{431}{20}$

(c) sum of elements in S_{20} is 589
(d) sum of elements in S_{20} is 609

19. If a, b and c are three terms of an A.P. such that $a \neq b$, then $\frac{b-c}{a-b}$ may be equal to

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 1 (d) 3

20. $(1^2 - a_1) + (2^2 - a_2)(3^2 - a_3) + ...(n^2 - a_n) = \frac{n}{3}(n^2 - 1)$ then
(a) $a_n = n$
(b) $a_1, a_2, a_3, ...$ are in A.P.
(c) $a_1, a_2, a_3, ...$ are in G.P.

(d) $\sum_{i=1}^{n} a_n = \frac{n(n+1)}{2}$

21. Consider the A.P. 50, 48, 46, 44, ... If S_n denotes sum to n terms of this A.P. then
(a) S_n is maximum for $n = 25$

(b) the first negative term is t_{26}
(c) the first negative term is t_{27}
(d) maximum value of S_n is 650.

22. For a positive integer n, let

$$S(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{2^n - 1}.$$ Then,

(a) $S_n \le n$ (b) $S_n > n$
(c) $S_{2n} \le n$ (d) $S_{2n} > n$

23. If the first and $(2n-1)^{\text{th}}$ terms of an A.P.; a G.P. and H.P. are equal and their n^{th} terms are p, q and s respectively, then which of the following options is/ are correct?

(a) $p \ge q \ge s$ (b) $p + s = q$
(c) $ps = q^2$ (d) $p = q = s$

24. Let f_1, f_2, f_3, ... be a sequence of integers satisfying $f_{n-1} + f_n = 2n \ \forall\ n \ge 2$ if $f_1 = 100$ then

(a) $f_{100} = 2$ (b) $f_{200} = 102$
(c) $f_{99} = 198$ (d) $f_9 = 100$

25. The sequence $\{a_n\}$; $n \ge 1$ satisfy $a_1 = 0$, $a_{n+1} = 5a_n + \sqrt{24a_n^2 + 1}$. Then

(a) a_n is rational for all n
(b) a_n is rational for $n \le 24$
(c) $a_{n+1} = 10a_n - a_{n-1}$
(d) a_n is positive integer for all $n \ge 2$

26. If a_1, a_2, a_3, ... are in A.P. with common difference d, $b_k = a_k + a_{k+1} \ldots + a_{k+n-1}$ $(k = 1, 2, 3, \ldots)$

(a) $\sum_{k=1}^{n} b_k = n^2 a_n$ (b) $\sum_{k=1}^{n} b_k = (n+1)^2 a_n$

(c) $b_k = \frac{n}{2}[a_n + a_1 + 2d(k-1)]$

(d) $\sum_{k=1}^{n} b_k = n(n+1)a_n$

27. Define $S_n = \frac{1}{\lfloor 3} + \frac{5}{\lfloor 4} + \frac{11}{\lfloor 5} + \ldots\ldots + \frac{n^2+n-1}{\lfloor n+2}$ then

(a) $t_{100} = \frac{10099}{\lfloor 102}$

(b) $S_{2009} = \frac{1}{2} - \frac{1}{\lfloor 2009} \cdot \frac{1}{2011}$

(c) $S_{2009} = \frac{1}{4} - \frac{1}{\lfloor 2009} \cdot \frac{1}{2011}$

(d) $\frac{1}{2}$

28. The cubes of the natural numbers are grouped as (1^3), $(2^3, 3^3)$, $(4^3, 5^3, 6^3)$ .....
Let S_n denote the sum of the cubes n^{th} group, then $8S_n$ is divisible by

(a) n^2 (b) n^3 (c) $n^2 + 1$ (d) $n^2 + 3$

29. Consider a sequence $\{a_n\}$ with $a_1 = 2$ and $a_n = \frac{a_{n-1}^2}{a_{n-2}}$ for all $n \ge 3$, terms of the sequence being distinct. Given that a_2 and a_5 are positive integers and $a_5 \le 162$, then the possible value(s) of a_5 can be

(a) 2 (b) 32 (c) 64 (d) 162

30. *a, b, c* are the first three terms of geometric series. If the H.M. of *a* and *b* is 12 and that of '*b*' and '*c*' is 36 then which of the following hold(s) good ?

(a) Sum of the first term and common ratio of the G.P. is 11.
(b) Sum of the first five terms of the G.P. is 948.
(c) If the value of the first term and common ratio of the given G.P. is taken as the first term and common difference of an A.P. then its 8^{th} term is 29.
(d) The number 648 is one of the term of the G.P.

31. Consider the sequence (a_n), given by

$$a_n = \frac{1000^n}{n!},\ n \in N$$

(a) $a_n \to \infty$ as $n \to \infty$ (b) $a_n \to 0$ as $n \to \infty$
(c) $a_n = a_{n+1}$ for exactly one value of n
(d) $a_n < a_{n+1}$ for all $n \in N$

32. Consider the A.P. 50, 48, 46, 44, ... If S_n denotes sum to n terms of this A.P. then

(a) S_n is maximum for $n = 25$
(b) the first negative term is t_{26}
(c) the first negative term is t_{27}
(d) maximum value of S_n is 650

SECTION-III

Comprehension Type

Paragraph for Question No. 33 to 35

Let a_1, a_2, a_3 be three numbers in arithmetic progression (A.P.) and g_1, g_2, g_3 be three numbers in geometric progression (G.P.). Also the values of $a_1 + g_1$, $a_2 + g_2$ and $a_3 + g_3$ are respectively 85, 76

and 84 and $\sum_{i=1}^{3} a_i = 126$.

33. The number of possible values that a_1 takes is
(a) 1 (b) 2 (c) 3 (d) 4

34. The sum of the common difference of the A.P. and the common ratio of the G.P. can be
(a) 27 (b) –24 (c) 31 (d) –33

35. The number of possible values that $\frac{a_2}{g_2}$ takes is
(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Question No. 36 to 38

Let $f(n)$ denotes the n^{th} term of the sequence 2, 5, 10, 17, 26, ... and $g(n)$ denotes the n^{th} term of the sequence 2, 6, 12, 20,.....
Let $F(n)$ and $G(n)$ respectively be the sum of n terms of the above sequences

36. $\lim_{n\to\infty} \frac{f(n)}{g(n)} =$
(a) 1 (b) 2
(c) 3 (d) doesn't exist

37. $\lim_{n\to\infty} \frac{F(n)}{G(n)} =$
(a) 0 (b) 1
(c) 2 (d) doesn't exist

38. $\lim_{n\to\infty}\left(\frac{F(n)}{G(n)}\right)^n - \lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right)^n$ is
(a) $\frac{\sqrt{e}-1}{e\sqrt{2}}$ (b) $\frac{\sqrt{e}+1}{e\sqrt{e}}$
(c) $\frac{1-\sqrt{e}}{e\sqrt{e}}$ (d) $\frac{e\sqrt{e}}{1+\sqrt{e}}$

Paragraph for Question No. 39 to 41

Two arithmetic progressions have the same number of terms. The ratio of the last term of the first progression to the first term of the second progression is equal to the ratio of the last term of the second progression to the first term of the first progression and is equal to 4. The ratio of the sum of the n terms of the first progression to the sum of n terms of the second progression is equal to 2.

39. Ratio of the first term of the first progression to the first term of the second progression is
(a) 8 (b) $\frac{2}{7}$ (c) 26 (d) $\frac{7}{2}$

40. Ratio of their common differences is
(a) 26 (b) $\frac{7}{2}$ (c) 2 (d) 8

41. Ratio of their last terms is
(a) $\frac{7}{54}$ (b) $\frac{54}{7}$ (c) $\frac{7}{2}$ (d) 26

Paragraph for Question No. 42 to 44

Natural numbers are divided into sets {1}, {2, 3}, {4, 5, 6}, {7, 8, 9, 10} .....
Now answer the following :

42. Sum of elements in 50^{th} set is
(a) 61064 (b) 62525
(c) 57625 (d) 60525

43. Last element in 2007^{th} set is
(a) 2015028 (b) 2107028
(c) 2510208 (d) 2370128

44. Average of middle two elements of 2008^{th} set is λ. Then $[\lambda]$ ..... where [.] is G.I.F
(a) 2106132 (b) 2016012
(c) 2016032 (d) 2160322

Paragraph for Question No. 45 to 47

We know that arithmetic mean of the positive numbers lie between them. Suppose $S_1 > S_2 > 0$ and $S_{n+1} = \frac{1}{2}(S_n + S_{n-1})$. We can easily conclude that S_3 lies between S_2 and S_1 and $S_2 < S_3 < S_1$

45. $S_{2n+1} - S_{2n}$ must be equal to
(a) $\left(\frac{1}{2}\right)^{2n-1}$ (b) $\left(\frac{1}{2}\right)^{2n-1}(S_1 - S_2)$
(c) $\left(\frac{1}{2}\right)^{2n-1}(S_2 - S_1)$ (d) zero

46. If $S_1 > S_2$, then $\lim_{n\to\infty} S_n$ must be equal to
(a) $S_1 - S_2$ (b) $S_1 + 2S_2$
(c) $\frac{S_1 + 2S_2}{3}$ (d) none of these

47. If $S_1 < S_2$, then the sequence $\{S_{2n}\}$ is
(a) increasing
(b) decreasing
(c) non-monotonic
(d) unbounded

SECTION-IV

Matrix-Match Type

48. Natural numbers are distributed into brackets {1}, {2, 3}, {4, 5, 6}, {7, 8, 9, 10},.....n^{th} bracket contains n elements. Now match the following :

	Column-I		Column-II
(A)	Sum of elements 50^{th} bracket is	(p)	2017037
(B)	First element of 2009^{th} bracket is	(q)	2016032
(C)	Last element of 2007^{th} bracket is	(r)	2015028
(D)	Average of middle two elements of 2008^{th} bracket is λ, then $[\lambda]$ is where [.] is G.I.F	(s)	62525

49. If $a + b + c = 1$ and (a, b, $c > 0$) then

	Column-I		Column-II
(A)	abc	(p)	≤ 64
(B)	$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$	(q)	$\le \frac{8}{27}$
(C)	$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)$	(r)	≤ 9
(D)	$(1-a)(1-b)(1-c)$	(s)	$\le \frac{1}{27}$

SECTION-V

Integer Answer Type

50. Let $a_1, a_2, \ldots\ldots, a_n$ be sequence of real numbers, with $a_{n+1} = a_n + \sqrt{1+a_n^2}$ and $a_0 = 0$ if $\lim_{n\to\infty} \frac{a_n}{2^{n-1}}$ is $\frac{k}{\pi}$, then k is equal to

51. Let $a_1 \neq 0$, and for $n > 1$; $a_n = \frac{n}{a_{n-1}}$ then exponent of 7 in $a_1a_2a_3 \ldots . a_{100}$ is

52. If $x = 2$ then the sum to infinity of the series $\frac{1}{x}+\frac{1}{2x^2}+\frac{1}{3x^3}+\ldots\ldots\ldots$ is $\frac{\log\lambda}{2}$, then λ =

53. Let T_n is n^{th} term of an A.P. If $\sum_{m=1}^{5^{99}} T_{2m} = 5^{100}$ and $\sum_{m=1}^{5^{99}} T_{2m-1} = 5^{99}$, then common difference of A.P. is

54. Number of natural numbers n satisfying the condition $[\sqrt{n+1}] \neq [\sqrt{n}]$ and $1 \le n \le 100$ is (where [.] denotes the greatest integer function)

55. Let $f(r)$ denote the number of zeros in $r!$. If $f(m) - f(n) = 3$, then value of $\frac{(m-n)_{max} - (m-n)_{min}}{2}$ is

56. The number of values of $n \in Z$ for which $n^2 + n + 2$ is perfect square is

57. If the roots of $x^5 - 40x^4 + px^3 + qx^2 + rx + S = 0$ are in G.P. and sum of their reciprocals is 10, then the value of $\sqrt{|2S|}$ is

58. Let $2^{a_1} + 3^{a_2} + 5^{a_3} + 7^{a_4} + 9^{a_5}$ is divisible by 4, where $a_1, a_2, \ldots\ldots, a_5$ are digits. If the number of solutions of $(a_1, a_2, a_3, a_4, a_5)$ is P, then $\frac{P}{9\times 5^4}$ is

59. Number of integral values of m satisfying $17^2 + n^4 = m^2$ is ($n \in I$)

60. If a, b, c are three positive real numbers, then minimum value of $\frac{a^2+1}{b+c}+\frac{b^2+1}{c+a}+\frac{c^2+1}{a+b}$ is

SOLUTIONS

1. (b) : $\frac{4000}{a_1a_{4001}} = 10 \Rightarrow a_1a_{4001} = 400$

$\left(\because \frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots+\frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}\right)$

$a_1 + a_{4001} = 50$

$(a_1 - a_{4001})^2 = (a_1 + a_{4001})^2 - 4a_1a_{4001}$

$= 2500 - 4 \times 400 = 900$

$\Rightarrow |a_1 - a_{4001}| = 30$

2. (c) : $p=\dfrac{x^{\ln y-\ln z}+y^{\ln z-\ln x}+z^{\ln x-\ln y}}{3}$

$\geq \sqrt[3]{x^{\ln y-\ln z}.y^{\ln z-\ln x}.z^{\ln x-\ln y}}$ using A.M. $\geq$ G.M. .....(1)

Let $\log x = a \Rightarrow x = e^a$

similarly $y = e^b$, $z = e^c$

from (1), we have

$\dfrac{e^{a(b-c)}+e^{b(c-a)}+e^{c(a-b)}}{3} \geq [e^{a(b-c)+b(c-a)+c(a-b)}]^{1/3} = 1$

$\Rightarrow \quad e^{a(b-c)}+e^{b(c-a)}+e^{c(a-b)} \geq 3$

$\therefore \quad x^{\log y-\log z}+y^{\log z-\log x}+z^{\log x-\log y} \in (3, \infty)$

3. (d) : Let n be a natural number greater than N^2. Consider $f(x)=\sqrt{x}$ defined on $[n, n+1]$. $f(x)$ satisfies condition of lagranges mean value theorem

$\Rightarrow \quad f'(c)=\dfrac{f(n+1)-f(n)}{n+1-n}$ for some $c \in (n, n+1)$ ..... (1)

and $f'(x)=\dfrac{1}{2\sqrt{x}} \Rightarrow f'(c)=\dfrac{1}{2\sqrt{c}}$ ..... (2)

$\dfrac{1}{2\sqrt{c}}=\sqrt{n+1}-\sqrt{n}$ (using (1) and (2))

$n>N^2 \Rightarrow c>N^2 \Rightarrow \dfrac{1}{2\sqrt{c}}<\dfrac{1}{2N}$

4. (b) : $S_n=\displaystyle\sum_{K=1}^{2008}\frac{K+2}{K!+(K+1)!+(K+2)!}$

$=\displaystyle\sum_{K=1}^{2008}\frac{K+2}{K!\ (K+2)^2}$

$=\displaystyle\sum_{K=1}^{2008}\frac{1}{K!(K+2)}=\sum_{K=1}^{2008}\frac{K+1}{(K+2)!}=\sum\frac{(K+2)-1}{(K+2)!}$

$=\displaystyle\sum_{K=1}^{2008}\left(\frac{1}{(K+1)!}-\frac{1}{(K+2)!}\right)$

Putting $K = 1, 2, 3, ..., 2008$

$\left(\dfrac{1}{2!}-\dfrac{1}{3!}\right)+\left(\dfrac{1}{3!}-\dfrac{1}{4!}\right)+\left(\dfrac{1}{4!}-\dfrac{1}{5!}\right)+.....$

$\left(\dfrac{1}{(2008)!}-\dfrac{1}{(2009)!}\right)+\left(\dfrac{1}{(2009)!}-\dfrac{1}{(2010)!}\right)$

$=\dfrac{1}{2}-\dfrac{1}{2010!}$

5. (c) : $\alpha_0 = 17.23$, $\alpha_1 = 33.23$

and $\alpha_{r+2}=\dfrac{\alpha_r+\alpha_{r+1}}{2}$

$\forall r \geq 0 \Rightarrow \alpha_{r+2}-\alpha_{r+1}=-\dfrac{1}{2}(\alpha_{r+1}-\alpha_r)$

$\alpha_{10}-\alpha_9=-\dfrac{1}{2}(\alpha_9-\alpha_8)=\dfrac{1}{2^2}(\alpha_8-\alpha_7)=.....$

$=-\dfrac{1}{2^9}(\alpha_1-\alpha_0)=-\dfrac{-1}{2^9}\times 16=\dfrac{-1}{2^5}$

$\therefore \quad |\alpha_{10}-\alpha_9|=\dfrac{1}{32}$

6. (a) : $\dfrac{\pi}{4}=1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\dfrac{1}{9}-\dfrac{1}{11}+.....$

$=\dfrac{1}{2}\left(\left(1-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{5}\right)+.....\right)=\dfrac{1}{2}.\dfrac{\pi}{4}=\dfrac{\pi}{8}$

7. (c) : $2n\ A_n = (2n-3)A_{n-1}$, $n \geq 2$

$2k\ A_k = (2k-3)A_{k-1}$, $k \geq 2$

$2k\ A_k - 2(k-1)A_{k-1} = -A_{k-1}$, $k \geq 2$ ..... (1)

Putting $k = 2, 3, 4, 5, ...n$ we have from (1)

$(4A_2 - 2A_1) + (6A_3 - 4A_2) + ... + (2nA_n - 2(n-1)A_{n-1}) = -(A_1 + A_2 + ... + A_{n-1})$

$\Rightarrow \quad -2A_1 + 2nA_n = -(A_1 + A_2 + ... + A_n) + A_n$

$\Rightarrow \quad -2A_1 + 2nA_n - A_n = -\displaystyle\sum_{k=1}^{n} A_k$

But from (1) we have $2n\ A_n - A_n = 2(n+1)A_{n+1}$

$\therefore \quad \displaystyle\sum_{k=1}^{n} A_k = 2A_1 - 2(n+1)A_{n+1}$

or $2\left(\dfrac{1}{2}\right) - 2(n+1)A_{n+1} = 1 - 2(n+1)A_{n+1}$

$\therefore \quad \displaystyle\sum_{k=1}^{n} A_k < 1, n \geq 1 \quad \because (n+1)A_{n+1} > 0$

8. (c) : Let $b_n=\dfrac{1}{a_n} \Rightarrow b_{n+1}+\dfrac{1}{3}=2\left(b_n+\dfrac{1}{3}\right)$ *i.e.* the sequence $< b_n+\dfrac{1}{3} >$ is a G.P with common ratio 2.

So $b_n+\dfrac{1}{3}=2^n\left(b_n+\dfrac{1}{3}\right) \Rightarrow b_n=\dfrac{1}{3}(2^{n+1}-1)$

Required $\displaystyle\sum_{i=1}^{n}\frac{1}{a_i}=b_0+b_1+b_2+....+b_n$

$$= \frac{1}{3}(2-1)+\frac{1}{3}(2^2-1)+...+\frac{1}{3}(2^{n+1}-1)$$

$$= \frac{1}{3}\left[2\left(\frac{2^{n+1}-1}{2-1}\right)-(n+1)\right] = \frac{1}{3}(2^{n+2}-n-3)$$

9. (d) : From A.M. ≥ G.M., we get

$$\frac{a_1+2a_2+3a_3+4a_4+5a_5}{5} \geq (a_1.2a_2.3a_3.4a_4.5a_5)^{1/5}$$

$$\Rightarrow \quad a_1+2a_2+3a_3+4a_4+5a_5 \geq 5(5!\ a_1a_2a_3a_4a_5)^{1/5}$$
$$= 5(6!)^{1/5}$$

10. (b) : Using the A.M. ≥ H.M. of $\frac{1}{a},\frac{1}{b},\frac{1}{c}$, we get

$$\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3} \geq \frac{3}{a+b+c} = \frac{3}{6} = \frac{1}{2} \quad \text{So, } \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{3}{2}$$

Now, $$\frac{\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2}{3}$$
$$\geq \left(\frac{a+\frac{1}{b}+b+\frac{1}{c}+c+\frac{1}{a}}{3}\right)^2 \geq \left(6+\frac{3}{2}\right)^2$$

$$= \left(\frac{5}{2}\right)^2 = \frac{25}{4} \ \therefore \ \left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2 \geq \frac{75}{4}$$

11. (a) : Let the first term and common difference of an A.P. be a_1 & x then

$a_6 = a_1 + 5x = 2 \Rightarrow a_1 = 2 - 5x$

Now $a_1 \cdot a_4 \cdot a_5 = (2-5x)(2-2x)(2-x)$

$\Rightarrow p = 8 - 32x + 34x^2 - 10x^3$

$$\Rightarrow \frac{dp}{dx} = -32 + 68x - 30x^2$$

for maxima, $\frac{dp}{dx} = 0 \Rightarrow x = \frac{2}{3}$ or $\frac{8}{5}$

but $\frac{dp^2}{dx^2} < 0$ for $x = \frac{8}{5}$

The common difference is $\frac{8}{5}$.

12. (a) : $$\frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+.... \Rightarrow \frac{S}{19} = \frac{4}{19^2}+\frac{44}{19^3}+....$$

$$\Rightarrow \quad S-\frac{S}{19} = \frac{4}{19}+\frac{40}{19^2}+\frac{400}{19^3}+.....$$

$$\Rightarrow \quad \frac{18S}{19} = \frac{4}{19}\left(1+\frac{10}{19}+\frac{100}{19^2}+.....\right)$$

$$\Rightarrow \quad \frac{18S}{19} = \frac{4}{19} \times \frac{1}{1-\frac{10}{19}} = \frac{4}{9} \Rightarrow S = \frac{38}{81}.$$

13. (b) : $s_n - s_{n-1} = t_n = 2c(n-1)$

$s = 4c^2\{0^2 + 1^2 + 2^2 + 3^2 + ...+ (n-1)^2\}$

$$= \frac{2}{3}c^2n(n-1)(2n-1)$$

14. (b) : $T_n = \frac{x^2}{x^3+200} = f(x) \Rightarrow f'(x) = \frac{x(400-x^3)}{(x^3+200)^2}$

$f(x)$ is increasing in $0 < x < 400^{1/3}$ and decreasing in $x > 400^{1/3}$ largest term T_7

$$T_7 = \frac{49}{543}$$

15. (a) : $$\left(\frac{a+b+c+d}{4}\right)^2 \leq \frac{a^2+b^2+c^2+d^2}{4}$$

$$\frac{(8-e)^2}{16} \leq \frac{16-e^2}{4} \Rightarrow 5e^2 - 16e \leq 0$$

$$\Rightarrow e(5e-16) \leq 0$$

$$\therefore \quad 0 \leq e \leq \frac{16}{5} \quad \therefore \quad e\left[0, \frac{16}{5}\right]$$

$\therefore \quad l + m + n = 21$

16. (b) : Let $A_1, A_2, ..., A_n$ be the n A.M.'s between a and $2b$. Let d be the common difference then

$$d = \frac{2b-a}{n+1}. \ m^{th} \text{ mean } A_m = a + \left(\frac{2b-a}{n+1}\right)m$$

If Am^1 is the m^{th} mean between $2a$ and b, then

$$Am^1 = 2a + \left(\frac{b-2a}{n+1}\right)m$$

Given $Am = Am^1 \Rightarrow \frac{a}{b} = \frac{m}{n-m+1}$

17. (b) : Let a, b be two numbers

$$A = \frac{a+b}{2}$$

$$G = \sqrt{ab}, H = \frac{2ab}{a+b}$$

$$xA = yG \Rightarrow \frac{x}{y} = \frac{2\sqrt{ab}}{a+b}$$

Also, $yG = zH \Rightarrow \frac{y}{z} = \frac{2\sqrt{ab}}{a+b}$

So, $y^2 = zx$, $x, y, z \in$ G.P.

18. (a, c) : The 1st elements of $S_1, S_2, S_3, S_4, \ldots$ are $\frac{1^2-1}{1}, \frac{2^2-1}{2}, \frac{3^2-1}{3}, \ldots$

$\Rightarrow$ S_i must be a set starting at $\frac{t^2-1}{i}$

$S_{20} = \left\{\frac{399}{20}, \frac{399+20}{20}, \frac{399+40}{20}, \ldots.20 \text{ terms}\right\}$

1st term of $S_{20} = \frac{399}{20}$

third term is $\frac{439}{20}$

$S_{20} = \frac{20}{2}\left[\frac{2\times 399}{20} + 19\times 1\right] = 589$

19. (c, d) : If a, b, c are p^{th}, q^{th} and r^{th} terms of an A.P. then $\frac{b-c}{a-b}$ is always a rational number.

As $a, b, c \in$ A.P.

$\therefore$ $2b = a + c$, which can be obtained only if $\frac{b-c}{a-b} = 1$

20. (a, b, d) : $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 - \{a_1 + a_2 + a_3 + \ldots\ldots + a_n\} = \frac{n(n^2-1)}{3}$

$\therefore$ $S_n = \frac{n(n+1)(2n+1)}{6} - \frac{n(n^2-1)}{3} = \frac{n^2+n}{2} = \frac{n(n+1)}{2}$

where $S_n = a_1 + a_2 + \ldots\ldots + a_n = \sum_{n=1}^{n} a_n$

$a_n = S_n - S_{n-1} = \frac{n(n+1)}{2} - \frac{(n-1)n}{2} = n.$

21. (a, c, d) : $t_n = 50 + (n-1)(-2) \le 0 \Rightarrow n \le 26$

$t_{26} = 0 \Rightarrow$ first negative term is t_{27}

S_n is maximum when $n = 25$ as $t_{26} = 0$

$S_n \max = S_{25} = \frac{25}{2}\{50 + 2\} = 650$ as $t_1 = 50$, $t_{25} = 2$

22. (a, d) : $S(n) = 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}\right) + \ldots + \left(\frac{1}{2^{n-1}} + \frac{1}{2^{n-1}+1} + \ldots + \frac{1}{2^n - 1}\right)$

$\le 1 + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \ldots + \left(\frac{1}{2^{n-1}} + \frac{1}{2^{n-1}} + \ldots + \frac{1}{2^{n-1}}\right)$

$= 1 + 1 + 1 + \ldots + 1$ (n terms) $= n$

Also $S(n) \ge 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \ldots + \left(\frac{1}{2^{n-2}+1} + \frac{1}{2^{n-2}+2} + \ldots + \frac{1}{2^{n-1}}\right)$

$> 1 + \frac{1}{2} + \frac{1}{2} + \ldots + \frac{1}{2} = 1 + \left(\frac{n-1}{2}\right) = \frac{n+1}{2}$

$\therefore$ $S(2n) > \frac{2n+1}{2} = n + \frac{1}{2} > n$

23. (a, c) : Let the first term be a and $(2n-1)^{\text{th}}$ term be b, then $t_{2n-1} = b$, $t_1 = a$

So, $b = a + (2n-2)d$

$\Rightarrow$ $d = \frac{b-a}{2n-2}$

$p = a + (n-1)d = a + (n-1)\left(\frac{b-a}{2n-2}\right) = \frac{a+b}{2}$

= A.M. of a and b

$\because$ $t_{2n-1} = ar^{2n-2} = b$ $\quad\therefore$ $r = \left(\frac{b}{a}\right)^{1/(2n-2)}$

$q = a \cdot r^{n-1} = a\left(\frac{b}{a}\right)^{\frac{n-1}{2n-2}} = a\left(\frac{b}{a}\right)^{\frac{1}{2}} = \sqrt{ab}$ = G.M. of a and b

$\Rightarrow$ $q^2 = ab$

$\frac{1}{s} = \frac{1}{a} + (n-1)\left(\frac{\frac{1}{b} - \frac{1}{a}}{2n-2}\right) = \frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{a+b}{2ab}$

$\therefore$ $S = \frac{2ab}{a+b}$ = H.M. of a and b

p, q, r are the A.M., G.M., H.M. of a and b.

$\therefore$ $p \ge q \ge r$ and $ps = q^2$

24. (a, b, c) : $f_{n-1} + f_n = 2n$ .....(1)

$f_{n-2} + f_{n-1} = 2n - 2$ .....(2)

$f_n - f_{n-2} = 2$

$\Rightarrow$ $\begin{cases} f_3 - f_1 = 2 \Rightarrow f_3 = 102 \text{ and} \\ f_2 + f_3 = 6 \Rightarrow f_2 = 6 - 102 = -96 \end{cases}$

sequence is 100 – 96, 102, ...

$f_n = \begin{cases} n-98, n \text{ even} \\ n+99, n \text{ odd} \end{cases}$

$\therefore\ f_{100} = 100 - 98 = 2, f_{200} = 200 - 98 = 102$ and $f_{99} = 99 + 99 = 198, f_9 = 9 + 99 = 108$

25. (a, b, c, d) : $a_{n+1}^2 - 10\,a_n a_{n+1} + a_n^2 - 1 = 0$

and $a_n^2 - 10\,a_n a_{n-1} + a_{n-1}^2 - 1 = 0$

Thus for $n \geq 2$, a_{n+1}, a_{n-1} are two distinct roots of $t^2 - 10t\,a_n + a_n^2 - 1 = 0$

$\therefore\ a_{n+1} + a_{n-1} = 10a_n \Rightarrow a_1 = 0, a_2 = 1, a_3 = 10$, etc...

26. (a, c) : $b_k = \frac{n}{2}\left[a_k + a_{n+k-1}\right]$

$= \frac{n}{2}\left[a_1 + (k-1)d + a_1 + (n+k-2)d\right]$

$= \frac{n}{2}[a_1 + (k-1)d + (n-1)d + (k-1)d]$

$= \frac{n}{2}[(a_1 + (n-1)d) + 2(k-1)d + a_1]$

$= \frac{n}{2}\left[a_n + a_1 + 2(k-1)d\right]$

$\sum_{k=1}^{n} b_k = \frac{n}{2}\left[na_n + na_1 + 2d\sum_{k=1}^{n}(k-1)\right]$

$= \frac{n}{2}\left[na_n + na_1 + \frac{2dn(n-1)}{2}\right]$

$= \frac{n^2}{2}\left[a_1 + (n-1)d + a_n\right]$

$= \frac{n^2}{2}\left[2a_n\right] = n^2 a_n$

27. (a, b) : $t_n = \frac{n^2+n-1}{\lfloor n+2} = \frac{n^2+2n-(n+1)}{\lfloor n+2}$

$= \left(\frac{1}{\lfloor n} - \frac{1}{\lfloor n+1}\right) - \left(\frac{1}{\lfloor n+1} - \frac{1}{\lfloor n+2}\right)$

$t_{100} = \frac{100^2+100-1}{\lfloor 102} = \frac{10099}{\lfloor 102}$

putting n = 1,2,3,... successively and adding

$S_n = \left[1 - \frac{1}{\lfloor n+1}\right] - \left[\frac{1}{2} - \frac{1}{\lfloor n+2}\right] = \frac{1}{2} - \frac{1}{\lfloor n+1} + \frac{1}{\lfloor n+2}$

28. (a, b, c, d) : Since r^{th} row contains r elements.

The number of elements in the first r rows

$= 1 + 2 + 3 + ... + r = \frac{1}{2}r(r+1) = K$(say)

$T_r = 1^3 + 2^3 + ... + K^3 = \frac{1}{4}K^2(K+1)^2$

and $T_{r-1} = \frac{r^2(r-1)^2(r^2-r+2)^2}{64}$

$= \frac{1}{64}r^2\left(r+1\right)^2 \cdot (r^2+r+2)^2$

Now that $S_n = T_n - T_{n-1} = \frac{1}{8}n^3(n^2+1)(n^2+3)$

$8S_n$ is divisible by n^2, n^3, n^2+1, n^2+3

29. (b, d) : Given $a_1 = 2;\ \frac{a_n}{a_{n-1}} = \frac{a_{n-1}}{a_{n-2}}$

$\Rightarrow$ a_1, a_2, a_3, a_4, a_5, ... in G.P.

Let $a_2 = x$ then for $n = 3$ we have

$\frac{a_3}{a_2} = \frac{a_2}{a_1} \Rightarrow a_2^2 = a_1 a_3 \Rightarrow a_3 = \frac{x^2}{2}\ (\because a_1 = 2 \text{ and } a_2 = x)$

$\therefore\ a_1\,a_2\,a_3 .... a_n = 2, x, \frac{x^2}{2}, \frac{x^3}{4}, \frac{x^4}{8}, ...$ with common ratio $r = \frac{x}{2}$

given $\frac{x^4}{8} \leq 162 \Rightarrow x^4 \leq 1296 \Rightarrow x \leq 6$

Also x and $\frac{x^4}{8}$ are integers

$\Rightarrow$ x must be even then only $\frac{x^4}{8}$ will be an integer.

Hence possible values of x is 4 and 6.
($x \neq 2$ as terms are distinct)

Hence possible value of $a_5 = \frac{x^4}{8}$ is $\frac{4^4}{8}, \frac{6^4}{8}$

$a_5 = 32, 162$

30. (a, c, d) : a, b, c are in G.P.; $b = ar$, $c = ar^2$

$12 = \frac{2ab}{a+b} \Rightarrow \frac{ab}{a+b} = 6 \Rightarrow \frac{a^2 r}{a(1+r)} = 6 \Rightarrow \frac{ar}{1+r} = 6$ ...(1)

again $36 = \frac{2bc}{b+c} \Rightarrow \frac{bc}{b+c} = 18$

$\frac{a^2 r^3}{ar(1+r)} = 18 \Rightarrow \frac{ar^r}{1+r} = 18$ ...(2)

from (1) and (2), $a = 8$, $r = 3$

Required G.P. = 8, 24, 72, 216, 648

(a) Sum of first term and common ratio = 8 + 3 = 11

(b) $S_5 = \frac{a(r^5-1)}{r-1} = 948$

(c) $t_1 = 8$, $d = 3 \Rightarrow$ t_8 of A.P. $= a + 7d = 29$

(d) t_5 of G.P. $= ar^4 = 648$

31. (b, c) : $a_n = \underbrace{\left(\frac{1000}{1}\cdot\frac{1000}{2}\cdots\frac{1000}{1000}\right)}_{k(\text{say})}\cdot\left(\frac{1000}{1001}\right)\left(\frac{1000}{1002}\right).....\left(\frac{1000}{n}\right),\ n>1000$

Thus $0 < a_n < k\left(\frac{1000}{1001}\right)^{n-1000}$, for $n > 1000$

Take n $\to \infty$ to realise that $a_n \to 0$, (b is true)

$a_n = a_{n+1} \Leftrightarrow \frac{1000^n}{n!} = \frac{1000^{n+1}}{(n+1)!}$

$\Leftrightarrow\ n + 1 = 1000$ i.e, $n = 999$

32. (a, c, d) : $t_n = 50 + (n-1)(-2) \le 0 \Rightarrow n \le 26$

$t_{26} = 0$ first negative term is t_{27}

S_n is maximum when $n = 25$ as $t_{26} = 0$

$S_n \max = S_{25} = \frac{25}{2}\{50 + 2\} = 650$

33. (b) **34. (b)** **35. (a)**

$a_1 + a_2 + a_3 = 126$

a_1, a_2, a_3 are in A.P. $\Rightarrow\ a_2 = \frac{126}{3} = 42$

The numbers in A.P. are $42 - d, 42, 42 + d$

The numbers in G.P. are $43 + d, 34, 42 - d$

A.P.plying the condition, we get $d = -26, 25$

The two possible sets of numbers are

1) A.P. 68, 42,16 G.P. 17, 34, 68

2) A.P. 17, 42, 67 G.P. 68, 34,17

(36-38) :

$f(n) = t_n$ of 2, 5, 10, 17, 26, ... $= n^2 + 1$

$\therefore\ F(n) = \Sigma t_n = \Sigma(n^2 + 1)$

$= \frac{n(n+1)(2n+1)}{6} + n = \frac{n(2n^2+3n+7)}{6}$

and $g(n) = T_n$ of 2, 6, 12, 20, ... $= n^2 + n$

$\therefore\ G(n) = \Sigma T_n = \Sigma(n^2 + n)$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{3}$

36. (a) : $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \lim_{n\to\infty}\frac{n^2+1}{n^2+n} = \lim_{n\to\infty}\frac{1+\frac{1}{n^2}}{1+\frac{1}{n}} = 1$

37. (b) : $\lim_{n\to\infty}\frac{F(n)}{G(n)} = \lim_{n\to\infty}\frac{\frac{n(2n^2+3n+7)}{6}}{\frac{n(n^2+3n+2)}{3}} = \frac{2}{6}\times\frac{3}{1} = 1$

38. (c) : $\lim_{n\to\infty}\left(\frac{F(n)}{G(n)}\right)^n - \lim_{n\to\infty}\frac{f(n)}{g(n)}$

$= \lim_{n\to\infty}\left(\frac{\frac{2n^3+3n^2+7n}{6}}{\frac{n^3+3n^2+2n}{3}}\right)^n - \lim_{n\to\infty}\left(\frac{n^2+1}{n^2+n}\right)^n$

$= \lim_{n\to\infty}\left(\frac{2n^3+3n^2+7n}{2n^3+6n^2+4n}\right)^n - \lim_{n\to\infty}\left(\frac{n^2+1}{n^2+n}\right)^n$

$= e^{\left(\lim_{n\to\infty}\frac{(-3n^2+3n)n}{n(2n^2+6n+4)}\right)} - e^{\lim_{n\to\infty}\left(\frac{n^2+1-n^2-c}{n^2+n}\right)n}$

$\therefore\ \lim_{n\to\infty}\left(\frac{f(x)}{g(x)}\right)^n$ is 1^∞ form $= e^{\lim_{n\to\infty}\left(\frac{f(n)-g(n)}{g(n)}\right)n}$

$= e^{-3/2} - e^{-1}$

$= \frac{e^{-1}(1-e^{1/2})}{e^{1/2}} = \frac{1-\sqrt{e}}{e\sqrt{e}}$

39. (b) **40. (a)** **41. (c)**

$\frac{t_n}{a_2} = \frac{t_n^1}{a_1} = 4 \Rightarrow \frac{t_n}{t_n^1} = \frac{a_2}{a_1}$

$t_n = a_1 + (n-1)d_1 = 4a_2$ ..... (1)

$t_n^1 = a_2 + (n-1)d_2 = 4a_1$ ..... (2) also $S_n = 2S_n^1$

i.e., $\frac{n}{2}[a_1 + t_n] = 2.\frac{n}{2}\left[a_2 + t_n^1\right]$

$\Rightarrow\ a_1 + 4a_2 = 2[a_2 + 4a_1] \Rightarrow\ 7a_1 = 2a_2 \Rightarrow \frac{a_1}{a_2} = \frac{2}{7}$

$1 + (n-1)\frac{d_1}{a_1} = 4\frac{a_2}{a_1}$ (from (1)) and

$$1+(n-1)\frac{d_2}{a_2}=4\frac{a_1}{a_2}=4.\frac{2}{7} \text{ (from (2))}$$

$$(n-1)\frac{d_1}{a_1}=4.\frac{7}{2}-1=13 \quad ...(3)$$

$$(n-1)\frac{d_2}{a_2}=\frac{8}{7}-1=\frac{1}{7} \quad ...(4)$$

Dividing (3) by (4), we get

$$\Rightarrow \frac{d_1}{d_2}.\frac{a_2}{a_1}=1\cdot3\cdot7 \Rightarrow \frac{d_1}{d_2}=1\cdot3\cdot7\cdot\frac{2}{7}=26$$

and $\frac{t_n}{t_n^1}=\frac{a_2}{a_1}=\frac{7}{2}$ (dividing (1) by (2))

(42-44) :

Given natural numbers are divided into sets

{1}, {2, 3} {4, 5, 6} {7, 8, 9, 10} {11, 12, 13, 14, 15} .....

Now the last term in the n^{th} set is the general term of the series of last element of each set and the series of last term of each set is 1, 3, 6, 10, 15, 21, ... whose n^{th} term is given by

$$t_n=a_1+(n-1)d_1+\frac{(n-1)(n-2)}{2!}d_2+....$$

$$=1+(n-1)(2)+\frac{(n-1)(n-2)}{2!}\cdot(1)+\frac{(n-1)(n-2)(n-3)}{3!}(0)$$

$$=\frac{1}{2}[2+4n-4+n^2-3n+2]=\frac{1}{2}[n^2+n]$$

$$=\frac{n(n+1)}{2} \text{ = last term of } n^{th} \text{ set or last element of } n^{th} \text{ set}$$

$\therefore$ last element of 50^{th} set $=\frac{50(51)}{2}=1275=l$

$\therefore$ first element of 50^{th} set = 1275 – 49 = 1226 = a

$\therefore$ $(S_n)_{n=50}=\frac{n}{2}(a+l)=\frac{50}{2}(1226+1275)$

= 25 × 2501 = 62525

OR

Last term (element) of n^{th} set $=\frac{n(n+1)}{2}$

and first term (element) of n^{th} set

$$=\frac{n(n+1)}{2}-(n-1)=\frac{n^2-n+2}{2}$$

$\therefore$ Sum of elements of n^{th} set which consists n number $=\frac{n}{2}\left[\frac{n(n+1)}{2}+\frac{n^2-n+2}{2}\right]$

$$=\frac{n}{4}[2n^2+2]=\frac{n}{2}(n^2+1)$$

42. (b)

$\therefore$ $S_{50}=\frac{50}{2}(50^2+1)=25(2501)=62525$

43. (a) : Last term of 2007^{th} set

$$=\frac{2007(2008)}{2}=2007\times1004=2015028$$

44. (c) : Last element of 2008^{th} set

$$=\frac{2008}{2}\times2009=1004\times2009=2017036$$

$\therefore$ first element of 2008^{th} set = 2017036 – 2007 = 2015029

$\therefore$ elements of 2008^{th} sets are = 2015029, 2015030, ..., 2017036

Two middle item (elements) are 1004^{th} and 1005^{th} terms

$\therefore$ 1004^{th} item of 2008^{th} set = 2015029 + 1003 = 2016032 and 1005^{th} item or element of $(2008)^{th}$ set = 2016033

$\therefore$ Average of two middle item

$$=\frac{2016032+2016033}{2}=2016032.5$$

$\therefore$ [2016032.5] = 2016032

45. (b) 46. (c) 47. (b)

(45-47)

Since, $S_3=\frac{S_1+S_2}{2}, S_2<S_3<S_1$

Again $S_4=\frac{S_2+S_3}{2} \Rightarrow S_2<S_4<S_3<S_1$

Further $S_5=\frac{S_4+S_3}{2} \Rightarrow S_4<S_5<S_3<S_1$

Continuing in this manner we will get

$S_2<S_4<S_6<S_8<.....<S_7<S_5<S_3<S_1$

$\Rightarrow$ sequence $\{S_{2n+1}\}$ is decreasing.

$$S_{2n+1}-S_{2n}=\frac{1}{2}(S_{2n}+S_{2n-1})-S_{2n}$$

$$= -\frac{1}{2}(S_{2n} + S_{2n-1})$$

$$= \left(-\frac{1}{2}\right)^2 (S_{2n-1} - S_{2n-2}) \text{ and so on.}$$

continuing in this manner, we get

$$S_{2n-1} + S_{2n-2} = \left(\frac{1}{2}\right)^{2-1} (S_1 - S_2)$$

(a) This recursion relation shows that this choice is not true.
(c) is ruled out because $S_{2n+1} - S_{2n}$ is always positive.
(d) is ruled out since S_{2n+1} is strictly greater than S_{2n}.

48. A – s; B – p, C – r; D – q

Sum of n^{th} bracket $= \frac{n}{2}(n^2+1)$

Last element of n^{th} bracket $= \frac{n(n+1)}{2}$

First element of $(n+1)^{th}$ bracket $= \frac{n(n+1)}{2} + 1$

49. A – s; B – r, C – p; D – q

We have $a + b + c = 1$

(A) A.M. $\geq$ G.M.

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc} \quad \Rightarrow \quad abc \leq \frac{1}{27}$$

(B) A.M. $\geq$ H.M.

$$\frac{a+b+c}{3} \geq \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}} \Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c} \leq 9$$

(C) $$\frac{\left(1+\frac{1}{a}\right)+\left(1+\frac{1}{b}\right)+\left(1+\frac{1}{c}\right)}{3} \geq \sqrt[3]{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)}$$

$$1+\frac{1}{3}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq \sqrt[3]{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)}$$

$$\Rightarrow \left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right) \leq \left(1+\frac{1}{3}\times 9\right)^3$$

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right) \leq 64$$

(D) $$\frac{(1-a)+(1-b)+(1-c)}{3} \geq \sqrt[3]{(1-a)(1-b)(1-c)}$$

$$\frac{3-1}{3} \geq \sqrt[3]{(1-a)(1-b)(1-c)}$$

$$(1-a)(1-b)(1-c) \leq \frac{8}{27}$$

50. (4) : $a_{n+1} = a_n + \sqrt{1+a_n^2}$ and $a_0 = 0$ ..... (1)

$\therefore \quad a_1 = a_0 + \sqrt{1+a_0^2} = 1$ .....(2)

Now assume $a_n = \cot\alpha_n$

$\therefore \quad a_{n+1} = \cot\alpha_n + \sqrt{1+\cot^2\alpha_n}$

$\Rightarrow \quad a_{n+1} = \cot\alpha_n + \operatorname{cosec}\alpha_n$

$$= \frac{\cos(\alpha_n)}{\sin(\alpha_n)} + \frac{1}{\sin(\alpha_n)} = \frac{2\cos^2\left(\frac{\alpha_n}{2}\right)}{2\sin\left(\frac{\alpha_n}{2}\right)\cos\left(\frac{\alpha_n}{2}\right)} = \cot\left(\frac{\alpha_n}{2}\right)$$

Also $a_n = \cot\alpha_n$

$\therefore \quad a_1 = \cot\alpha_1 = 1$ using (2)

$\alpha_1 = \pi/4$

Again $a_{n+1} = a_n + \sqrt{1+a_n^2}$

$\therefore a_2 = a_1 + \sqrt{1+a_1^2} = 1+\sqrt{2}$ using (1) by putting $n = 1$

$$a_2 = \sqrt{2}+1 = \cot\left(22\frac{1}{2}\right)^\circ = \cot\left(\frac{\pi}{4.2}\right)$$

Similarly $a_3 = \cot\left(\frac{\pi}{4\times 2^2}\right)$

$$a_n = \cot\left(\frac{\pi}{4\cdot 2^{n+1}}\right)$$

$$\therefore \lim_{n\to\infty} \frac{a_n}{2^{n+1}} = \lim_{n\to 0} \frac{\cot\left(\frac{\pi}{4.2^{n-1}}\right)}{2^{n-1}}$$

$$\therefore \lim_{X\to 0} = \frac{\cot\left(\frac{\pi}{4\cdot 2^{n-1}}\right)}{2^{n-1}} \quad \text{Putting } X = \frac{1}{2^{n-1}}$$

$$= \lim_{X\to 0} = \frac{1}{\frac{\tan\left(\frac{\pi X}{4}\right)}{X}} \quad \text{as } n\to\infty \Rightarrow X\to 0$$

$$= \lim_{X\to 0} = \frac{1}{\frac{\tan\frac{\pi X}{4}}{\frac{\pi}{4}X}\times\frac{\pi}{4}} = \frac{4}{\pi} = k$$

$\therefore \quad k = 4$

51. (8) : $a_{n-1}.\ a_n = n$

taking $n = 2, 4, 6,8, \ldots.. 100$, we get

$a_1a_2.a_3a_4 \ldots\ldots\ldots a_{99}a_{100} = 2{\cdot}4{\cdot}6{\cdot}8\ldots\ldots 100 = 2(50)!$

Exponent of 7 in 2(50)! = exponent of 7 in 50!

$$\left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right] = 7+1 = 8, \text{ where } [.] \text{ denotes greatest integer function.}$$

52. (4) : $\log\left(1-\frac{1}{x}\right) = \frac{-1}{x} - \frac{1}{2x^2} - \frac{1}{3x^3}\ldots\ldots$

$$\frac{1}{2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}+\ldots\ldots = -\log\left(1-\frac{1}{2}\right) = \log 2 = \frac{1}{2}\log 4$$

$\therefore \quad \lambda = 4$

53. (8) : $\sum_{m=1}^{5^{99}} T_{2m} = \sum_{m=1}^{5^{99}} (T_{2m-1} + d)$

(where d = common difference of A.P.)

$$= 5^{100} = \sum_{m=1}^{5^{99}} T_{2m-1} + \sum_{m=1}^{5^{99}} d$$

$$\Rightarrow \quad 5^{100} = 5^{99} + (5^{99})d$$

$$\Rightarrow \quad d = \frac{5^{100} - 5^{99}}{5^{99}} = \frac{5^{99}(5-1)}{5^{99}} = 4$$

clearly 3^3 is a factor of x and 3^2 is a factor of atleast one of y and z. This can be done in 5 ways.

Also 5^3 is a factor of atleast two of the numbers x, y, z which can be done in $({}^3C_2 \times 4 - 2) = 10$.

54. (9) : $\left[\sqrt{n+1}\right] \neq \left[\sqrt{n}\right]$

$$0 < \sqrt{n+1} - \sqrt{n} < 1 \qquad \ldots\ldots(1)$$

$$\Rightarrow \quad \left[\sqrt{n+1}\right] = \left[\sqrt{n}\right] \text{ or } \left[\sqrt{n}\right]+1 \qquad \ldots\ldots(2)$$

from (1) and (2), we get

$$\left[\sqrt{n+1}\right] = \left[\sqrt{n}\right]+1$$

$$k^2 \le n+1 < (k+1)^2 \Rightarrow \left[\sqrt{n+1}\right] = k$$

$$\Rightarrow \quad n \ge k^2 - 1 \qquad \ldots\ldots(3)$$

$$\left[\sqrt{n}\right] = k-1 \Rightarrow n < k^2 \qquad \ldots\ldots(4)$$

from (3) and (4) $n = k^2 - 1$ and $1 \le n \le 100$ (given)

$1 \le k^2 - 1 \le 100 \ \Rightarrow \ k = 2, 3, \ldots\ldots, 10.$

No. of values of $k = 9$

55. (9) : $f(m) - f(n) = 3$

if $m = 125, n = 124 \quad \Rightarrow (m-n)_{\min} = 1$

$m = 24, n = 5 \ \Rightarrow \ f(m) - f(n) = 3$

$\Rightarrow \quad (m-n)_{\max} = 19.$

$$\Rightarrow \quad \frac{(m-n)_{\max} - (m-n)_{\min}}{2} = 9$$

56. (2) : $n^2 + n + 2 = m^2$ or $4(n^2 + n + 2) = 4m^2$

$\Rightarrow \ (2m)^2 - (2n+1)^2 = 7 \ \Rightarrow 4(m^2 - n^2 - n) - 1 = 7$

$(2n+1)^2 = 9 \ \Rightarrow \ 2n + 1 = \pm 3 \ \Rightarrow n = 1, -2.$

57. (8) : Let roots be $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

then $\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2 = 40 \qquad \ldots\ldots(1)$

and $\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2} = 10 \qquad \ldots\ldots(2)$

$$\frac{a\left[\frac{1}{r^2}+\frac{1}{r}+1+r+r^2\right]}{\frac{1}{a}\left[r^2+r+1+\frac{1}{r}+\frac{1}{r^2}\right]} = \frac{40}{10} \quad \text{[from (1) and (2)]}$$

$a^2 = 4 \ \Rightarrow \ a = \pm 2$

$$\text{Product of roots} = \frac{a}{r^2}\times\frac{a}{r}\times a\times ar\times ar^2 = -S$$

$a^5 = -S \ \Rightarrow \ S = -a^5 = \pm 32 \Rightarrow S = -32$

$\Rightarrow |2S| = 64 \ \Rightarrow \ \sqrt{2S} = \sqrt{|64|} = 8$

58. (8) : Possible cases are if $a_1 = 0$, no way that the number can be divisible by 4.

If $a_1 = 1$, out of a_2 and a_4 one has to be even, one odd and a_3 and a_5 can be any.

If $a_1 \ge 2$, a_2 and a_4 both should be either even or odd and a_3 and a_5 can be any.

59. (4) : $17^2 = (m - n^2)(m + n^2)$

$m + n^2 = 17^2 \qquad m + n^2 = 17$

$m - n^2 = 1 \qquad m - n^2 = 17$

there are 4 such values of m possible.

60. (3) : Using the fact

$$a^2+1 \geq 2a \Rightarrow \frac{a^2+1}{b+c} \geq \frac{2a}{b+c} \quad \text{....(1)}$$

$$b^2+1 \geq 2b \Rightarrow \frac{b^2+1}{c+a} \geq \frac{2b}{c+a} \quad \text{....(2)}$$

$$c^2+1 \geq 2c \Rightarrow \frac{c^2+1}{a+b} \geq \frac{2c}{a+b} \quad \text{....(3)}$$

$\therefore$ from (1) (2) & (3) we have

$$\sum \frac{a^2+1}{b+c} \geq \frac{2a}{b+c}+\frac{2b}{c+a}+\frac{2c}{a+b}$$

Using fact A.M. of $(-1)^{th}$ powers $\geq (-1)^{th}$ power of A.M.

$$\therefore \quad \frac{\left(\frac{b+c}{a+b+c}\right)^{-1}+\left(\frac{c+a}{a+b+c}\right)^{-1}+\left(\frac{a+b}{a+b+c}\right)^{-1}}{3} \geq \left(\frac{\frac{b+c}{a+b+c}+\frac{c+a}{a+b+c}+\frac{a+b}{a+b+c}}{3}\right)^{-1}$$

$$\Rightarrow \quad \frac{\frac{a+b+c}{b+c}+\frac{a+b+c}{c+a}+\frac{a+b+c}{a+b}}{3} \geq \left(\frac{2}{3}\right)^{-1}$$

$$\Rightarrow \quad \left(\frac{a}{b+c}+1\right)+\left(\frac{b}{a+c}+1\right)+\left(\frac{c}{a+b}+1\right) \geq \frac{9}{2}$$

$$\Rightarrow \quad \frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \geq \frac{9}{2}-3=\frac{3}{2}$$

$$\Rightarrow \quad \frac{2a}{b+c}+\frac{2b}{c+a}+\frac{2c}{a+b} \geq 3$$

Now $\frac{a^2+1}{b+c}+\frac{b^2+1}{c+a}+\frac{c^2+1}{a+b} \geq \frac{2a}{b+c}+\frac{2b}{c+a}+\frac{2c}{a+b} \geq 3$

$\therefore$ Minimum of $\sum \frac{a^2+1}{b+c}=3$

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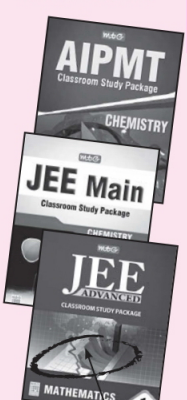

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Class XII

Matrices and Determinants

* ALOK KUMAR, B.Tech, IIT Kanpur

This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

DEFINITION

- Rectangular array of mn numbers. Unlike determinants it has no value.

$$A=\begin{bmatrix} a_{11} & a_{12} & ..... & a_{1n} \\ a_{21} & a_{22} & ..... & a_{2n} \\ : & : & : & : \\ a_{m1} & a_{m2} & ..... & a_{mn} \end{bmatrix}$$

or

$$\begin{pmatrix} a_{11} & a_{12} & ..... & a_{1n} \\ a_{21} & a_{22} & ..... & a_{2n} \\ : & : & : & : \\ a_{m1} & a_{m2} & ..... & a_{mn} \end{pmatrix}$$

Abbreviated as : $A = [a_{ij}], 1 \le i \le m; 1 \le j \le n$, where i denotes the row and j denotes the column, is called a matrix of order $m \times n$.

Special Type Of Matrices

- **Row Matrix (or row vectors) :** $A = [a_{11}, a_{12}, ... a_{1n}]$ having one row called $(1 \times n)$ matrix.
- **Column Matrix (or column vectors) :** $A=\begin{bmatrix} a_{11} \\ a_{21} \\ : \\ a_{m1} \end{bmatrix}$ having one column called $(m \times 1)$ matrix
- **Zero or Null Matrix : ($A = O_{m \times n}$)**
An $m \times n$ matrix all whose entries are zero.

$$A=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$ is 3×2 null matrix

and $B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is 3×3 null matrix.

- **Horizontal Matrix :** A matrix of order $m \times n$ is a horizontal matrix if $n > m$.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 1 & 1 \end{bmatrix}$$

- **Vertical Matrix :** A matrix of order $m \times n$ is a vertical matrix if $m > n$.

$$\begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 6 \\ 2 & 4 \end{bmatrix}$$

- **Square Matrix : (Order n)**
If number of rows = number of columns
$\Rightarrow$ a square matrix.

REMARKS :

- In a square matrix the pair of elements a_{ij} & a_{ji} are called **Conjugate Elements.**

$$e.g.\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

- The elements a_{11}, a_{22}, a_{33}, ..., a_{nn} are called **Diagonal Elements.** The line along which the diagonal elements lie is called **"Principal or Leading"** diagonal.

The sum of all diagonal entries $\sum_{i=j=1}^{n} a_{ij}$ = trace of the matrix written as, tr$A$

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

Square Matrix

Triangular Matrix

$$A=\begin{pmatrix}1 & 3 & -2\\ 0 & 2 & 4\\ 0 & 0 & 5\end{pmatrix};\qquad B=\begin{pmatrix}1 & 0 & 0\\ 2 & -3 & 0\\ 4 & 3 & 3\end{pmatrix}$$

Upper Triangular $a_{ij}=0, \forall\, i>j$

Lower Triangular $a_{ij}=0, \forall\, i<j$

Diagonal Matrix is denoted as $d_{diag}\,(d_1, d_2, \ldots\ldots, d_n)$ all elements except the leading diagonal are zero.

Diagonal Matrix

$$\begin{bmatrix}d_1 & 0 & 0\\ 0 & d_2 & 0\\ 0 & 0 & d_3\end{bmatrix}$$

Unit or Identity Matirx

$$a_{ij}=\begin{bmatrix}1, & \text{if} & i=j\\ 0, & \text{if} & i\neq j\end{bmatrix}$$

Note that : Minimum number of zeros in a triangular matrix of order $n = n(n-1)/2$

Note : (1) If $d_1 = d_2 = d_3 = a$ Scalar Matrix
(2) If $d_1 = d_2 = d_3 = 1$ Unit Matrix

Note : Minimum number of zeros in a diagonal matrix of order $n = n(n-1)$
"It is to be noted that with square matrix there is a corresponding determinant formed by the elements of A in the same order."

Equality Of Matrices :

Let $A = [a_{ij}]$ & $B = [b_{ij}]$ are equal if

- both have the same order .
- $a_{ij} = b_{ij}$ for each pair of i and j.

Algebra Of Matrices :

Addition : $A + B = [a_{ij} + b_{ij}]$ where A & B are of the same type. (same order)

- **Matrix addition is associative .**
  $(A + B) + C = A + (B + C)$
  where A, B and C are of the same type.
- **Addition of matrices is commutative.**
  i.e. $A + B = B + A$
  where A and B are of the same order.
- **Additive inverse**
  If $A + B = O = B + A$, $A = m \times n$
- **Multiplication Of A Matrix By A Scalar :**

$$A=\begin{bmatrix}a & b & c\\ b & c & a\\ c & a & b\end{bmatrix};\ kA=\begin{bmatrix}ka & kb & kc\\ kb & kc & ka\\ kc & ka & kb\end{bmatrix}$$

- **Multiplication of Matrices : (Row by Column)**
  AB exists if , $O(A) = m \times n$ & $O(B) = n \times p$
  AB exists, but BA does not $\Rightarrow AB \neq BA$

Note : In the product AB, $\begin{cases}A = \text{pre factor}\\ B = \text{post factor}\end{cases}$

$$A=(a_1, a_2, \ldots\ldots a_n)_{1\times n}\ \&\ B=\begin{bmatrix}b_1\\ b_2\\ \vdots\\ b_n\end{bmatrix}_{n\times 1}$$

$AB = [a_1b_1 + a_2b_2 \ldots\ldots a_nb_n]_{1\times 1}$

If $A = [a_{ij}]_{m\times n}$ & $B = [b_{ij}]_{n\times p}$ matrix, then

$$(AB)_{ij}=\sum_{r=1}^{n} a_{ir}.b_{rj}$$

Properties Of Matrix Multiplication :

- Matrix multiplication is not commutative .

$$A=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix};\ B=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix};\ AB=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix};\ BA=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$$

$\Rightarrow AB \neq BA$ (in general)

$$AB = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow AB = O \not\Rightarrow A = O$ or $B = O$

Note : If A and B are two non- zero matrices such that $AB = O$ then A and B are called the divisors of zero. Also if $[AB] = O \Rightarrow |AB| \Rightarrow |A|\,|B| = 0$

$\Rightarrow |A| = 0$ or $|B| = 0$ but not the converse.

If A and B are two matrices such that

(i) $AB = BA \Rightarrow A$ and B commute each other

(ii) $AB = -BA \Rightarrow A$ and B anti-commute each other

Matrix Multiplication Is Associative :

If A, B & C are conformable for the product AB & BC, then $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Distributivity :

$$\left.\begin{aligned} A\,(B + C) &= AB + AC \\ (A + B)\,C &= AC + BC \end{aligned}\right]$$

Provided A, B and C are conformable for respective products.

Positive Integral Powers Of A Square Matrix :

For a square matrix A,

$A^2A = (AA)A = A(AA) = A^3$.

Note that for a unit matrix I of any order, $I^m = I$ for all $m \in N$.

Matrix Polynomial :

If $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots\ldots + a_nx^0$ then we define a matrix polynomial $f(A) = a_0A^n + a_1A^{n-1} + a_2A^{n-2} + \ldots\ldots + a_nI^n$ where A is the given square matrix. If $f(A)$ is the null matrix then A is called the zero or root of the polynomial $f(x)$.

Definitions :

- **Idempotent Matrix :** A square matrix is idempotent provided $A^2 = A$.
  Note : $A^n = A \ \forall \ n \in N$.
- **Nilpotent Matrix :** A square matrix is said to be nilpotent matrix of order m, $m \in N$, if $A^m = O$, $A^{m-1} \neq O$.
- **Periodic Matrix :** A square matrix which satisfies the relation $A^{k+1} = A$, for some positive integer k, is a periodic matrix. The period of the matrix is the least value of k for which this holds true.
  Note : Period of an idempotent matrix is 1.
- **Involutory Matrix :** If $A^2 = I$, then matrix is said to be an involutory matrix.
  Note : $A = A^{-1}$ for an involutory matrix.

The Transpose Of A Matrix :

Let A be any matrix . Then , $A = a_{ij}$ of order $m \times n$

$\Rightarrow A^T$ or $A' = [a_{ji}]$ for $1 \le i \le n$ and $1 \le j \le m$ of order $n \times m$

Properties of Transpose : If A^T & B^T denote the transpose of A and B,

(a) $(A \pm B)^T = A^T \pm B^T$; note that A and B have the same order.

(b) $(AB)^T = B^T A^T$, A and B are conformable for matrix product AB.

(c) $(A^T)^T = A$

(d) $(kA)^T = kA^T$, k is a scalar.

Note : $(A_1 \cdot A_2 \cdot \ldots\ldots A_n)^T = A_n^T \cdot \ldots\ldots, A_2^T \cdot A_1^T$ (reversal law for transpose)

Symmetric and Skew Symmetric Matrix :

A square matrix $A = [a_{ij}]$ is said to be **symmetric** if, $a_{ij} = a_{ji} \ \forall \ i$ & j (conjugate elements are equal) (Note $A = A^T$)

Remark : Maximum number of distinct entries in a symmetric matrix of order n is $\frac{n(n+1)}{2}$.

A square matrix is said to be **skew symmetric** if, $a_{ij} = -a_{ji} \ \forall \ i$ & j (the pair of conjugate elements are additive inverse of each other) (Note $A = -A^T$)

Hence, if A is skew symmetric, then

$a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0 \ \forall \ i$

Thus the digaonal elements of a skew symmetric matrix are all zero , but not the converse.

Properties Of Symmetric and Skew Symmetric Matrix :

- A is symmetric if $A^T = A$
- A is skew symmetric if $A^T = -A$
- $A + A^T$ is a symmetric matrix
- $A - A^T$ is a skew symmetric matrix .

Consider $(A + A^T)^T = A^T + (A^T)^T = A^T + A$
$= A + A^T$

$\Rightarrow A + A^T$ is symmetric.

Similarly we can prove that $A - A^T$ is skew symmetric.

- The sum of two symmetric matrix is a symmetric matrix and the sum of two skew symmetric matrix is a skew symmetric matrix.

Let $A^T = A$; $B^T = B$, where A and B have the same order .

$(A + B)^T = A + B$

Similarly we can prove the other

- If A and B are symmetric matrices, then

(a) $AB + BA$ is a symmetric matrix.

(b) $AB - BA$ is a skew symmetric matrix.

Every square matrix can be uniquely expressed as a sum of a symmetric and a skew symmetric matrix.

$$A = \frac{1}{2}\underbrace{(A + A^T)}_{P} + \frac{1}{2}\underbrace{(A - A^T)}_{Q}$$

P: Symmetric; Q: Skew Symmetric

Adjoint Of A Square Matrix :

Let $A = [a_{ij}] = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a square matrix and let the matrix formed by the cofactors of $[a_{ij}]$ in determinant $|A|$ is

$$= \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$$

Then $(\text{adj } A) = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$

Note : $A(\text{adj}A) = (\text{adj } A)A = |A|I_n$, if A be a square matrix of order n.

REMARKS :

If A and B are non-singular square matrices of same order, then

- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$
- $\text{adj}(KA) = K^{n-1}(\text{adj } A)$, K is a scalar

Inverse Of A Matrix (Reciprocal Matrix) :

A square matrix A said to be invertible (non singular) if there exists a matrix B such that,

$AB = I = BA$

B is called the inverse (reciprocal) of A and is denoted by A^{-1}. Thus

$A^{-1} = B \Leftrightarrow AB = I = BA$.

We have, $A(\text{adj } A) = |A| I_n$

$A^{-1} A (\text{adj } A) = A^{-1} I_n |A|$

$I_n (\text{adj } A) = A^{-1} |A| I_n$

$\therefore \quad A^{-1} = \dfrac{(\text{adj } A)}{|A|}$

Note : The necessary and sufficient condition for a square matrix A to be invertible is that $|A| \neq 0$.

If A & B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1} A^{-1}$. This is reversal law for inverse.

REMARKS :

- If A be an invertible matrix, then A^T is also invertible & $(A^T)^{-1} = (A^{-1})^T$.
- If A is invertible, **(a)** $(A^{-1})^{-1} = A$;
  (b) $(A^k)^{-1} = (A^{-1})^k = A^{-k}$, $k \in N$
- If A is an orthogonal matrix, then $AA^T = I = A^TA$
- A square matrix is said to be orthogonal if $A^{-1} = A^T$.

$\therefore \quad |A^{-1}| = \dfrac{1}{|A|}$

System Of Equations and Criterion For Consistency

GAUSS - JORDAN METHOD

$x + y + z = 6$

$x - y + z = 2$

$2x + y - z = 1$

or $\begin{pmatrix} x+y+z \\ x-y+z \\ 2x+y-z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$

$AX = B \Rightarrow A^{-1}AX = A^{-1}B$, where $|A| \neq 0$

$$X = A^{-1}B = \frac{(\text{adj } A)\cdot B}{|A|}$$

REMARKS :

- If $|A| \neq 0$, system is consistent having unique solution
- If $|A| = 0$ & $(\text{adj } A)\cdot B = O$ (Null matrix), system is consistent having infinitely many solutions.
- If $|A| = 0$ & $(\text{adj } A)\cdot B \neq O$ (Null matrix), system is consistent having no solution.
- If $|A| = 0$, **matrix method fails**

If $(\text{adj } A)B$ = null matrix $= O$ → Consistent (Infinite solutions)

If $(\text{adj } A)B \neq O$ → Inconsistent (no solution)

- The symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called the determinant of order two.
  Its value is given by : $D = a_1b_2 - a_2b_1$
- The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is called the determinant of order three.
  Its value can be found as :

$$D = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$D = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

..... and so on.
In this manner we can expand a determinant in 6 ways using elements of
R_1, R_2, R_3 or C_1, C_2, C_3

- Following examples of short hand writing of large expressions are :
- Equation of a straight line passsing through (x_1, y_1) and (x_2, y_2) is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

- The lines : $a_1x + b_1y + c_1 = 0$ ..... (1)
  $a_2x + b_2y + c_2 = 0$ ..... (2)
  $a_3x + b_3y + c_3 = 0$ ..... (3)
  are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$
  Condition for the consistency of three simultaneous linear equations in two variables.
- Area of a triangle whose vertices are (x_r, y_r); $r = 1, 2, 3$ is :

$$D = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If $D = 0$ then the three points are collinear.

- $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

Minors

The minor of a given element of a determinant is the determinant of the elements which remain after deleting the row and the column in which the given element stands. For example,

Let the determinant be $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then minor of a_1 is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ and the minor of b_2 is $\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}$.

Hence a determinant of order two will have "4 minors" & determinant of order three will have "9 minors".

Cofactor

If M_{ij} represents the minor of some typical element then the cofactor is defined as :
$C_{ij} = (-1)^{i+j}.M_{ij}$; where i & j denotes the row & column in which the particular element lies.

Note : The value of a determinant of order three in terms of 'Minor' & 'Cofactor' can be written as : $D = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ or $D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$ & so on .....

Properties Of Determinants :

- The value of a determinant remains unaltered, if the rows & columns are interchanged .

$$e.g.\ \text{if } D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D'$$

D & D' are transpose of each other. If $D' = -D$ then it is skew symmetric determinant but $D' = D \Rightarrow 2D = 0 \Rightarrow D = 0$

$\Rightarrow$ Skew symmetric determinant of third order has the value zero.

- If any two rows (or columns) of a determinant be interchanged, the value of determinant is changed in sign only. e.g.

$$\text{Let } D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ \& } D'=\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Then $D' = -D$.

- If a determinant has any two rows (or columns) identical, then its value is zero.

$$\text{e.g. Let } D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ then it can be verified}$$

that $D = 0$.

- If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

$$e.g.\ \text{If } D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ \& } D'=\begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Then $D' = KD$

- If each element of any row (or column) can be expressed as a sum of two terms then the determinant can be expressed as the sum of two determinants . e.g.

$$\begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

- The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column).

$$e.g.\ \text{Let } D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and}$$

$$D'=\begin{vmatrix} a_1+ma_2 & b_1+mb_2 & c_1+mc_2 \\ a_2 & b_2 & c_2 \\ a_3+na_1 & b_3+nb_1 & c_3+nc_1 \end{vmatrix}$$

Then $D' = D$.

Note : that while applying this property atleast one row (or column) must remain unchanged.

If by putting $x = a$ the value of a determinant vanishes then $(x - a)$ is a factor of the determinant.

Multiplication Of Two Determinants :

- $$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1l_1+b_1l_2 & a_1m_1+b_1m_2 \\ a_2l_1+b_2l_2 & a_2m_1+b_2m_2 \end{vmatrix}$$

Similarly two determinants of order three are multiplied.

- If $D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

then, $D^2=\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

where A_i, B_i, C_i are cofactors

Consider

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{vmatrix}$$

where, $a_1A_2 + b_1B_2 + c_1C_2 = 0$ etc.

therefore, $D \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^3$

$$\Rightarrow \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = D^2 \text{ or } \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = D^2$$

System Of Linear Equations (In Two Variables) :

- **Consistent Equations :** If the equation has at least one solution
  - **(i) Independent equation :** Unique solution [Intersecting lines]
  - **(ii) Dependent equation :** Infinite solutions. [Identical lines]
- **Inconsistent Equation :** No solution. [Parallel lines]

  Let $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

  then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

  $\Rightarrow$ Given equations are inconsistent &

  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

  $\Rightarrow$ Given equations are dependent

Cramer's Rule : [Simultaneous Equations Involving Three Unknowns]

Let $a_1x + b_1y + c_1z = d_1$ ...(I)

$a_2x + b_2y + c_2z = d_2$ ...(II)

$a_3x + b_3y + c_3z = d_3$ ...(III)

Then, $x = \frac{D_1}{D}, y = \frac{D_2}{D}, z = \frac{D_3}{D}$

Where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$; $D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$;

$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ & $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Remarks :

- If $D \neq 0$ and atleast one of $D_1, D_2, D_3 \neq 0$, then the given system of equations are consistent and have unique non-trivial solution.
- If $D \neq 0$ & $D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have trivial solution only i.e., $x = y = z = 0$
- If $D = D_1 = D_2 = D_3 = 0$, then the given system of equations are consistent and have infinite solutions.

  In case $\left.\begin{aligned} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{aligned}\right\}$ represents these parallel planes then also

  $D = D_1 = D_2 = D_3 = 0$ but the system is inconsistent.
- If $D = 0$ but atleast one of D_1, D_2, D_3 is not zero then the equations are inconsistent and have no solution.
- If x, y, z are not all zero, the condition for $a_1x + b_1y + c_1z = 0$; $a_2x + b_2y + c_2z = 0$ & $a_3x + b_3y + c_3z = 0$ to be consistent in x, y, z is that $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Remember that if a given system of linear equations have only zero solution for all its variables then the given equations are said to have Trivial solution i.e. if $x = y = z = 0$, then the system of equations are consistent and solution is called trivial solution.

SECTION-I

Single Correct Answer Type

1. If $D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$ then $\sum_{r=1}^{n} D_r =$

(a) 0 (b) -1 (c) 1 (d) n

2. The system of linear equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$ has no solution when

(a) $a = 2, b \neq 3$ (b) $b = 2, a = 3$

(c) $a = 3, b \neq 10$ (d) $b = 3, a \neq 10$

3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A^4 - 5A^3 - A^2 - 4A - 2I =$

(a) 0 (b) A (c) I (d) $2I$

4. If α, β, γ are the real roots of a cubical polynomial satisfying $\begin{vmatrix} \alpha^2 & \beta^2 & \gamma^2 \\ (\alpha+1)^2 & (\beta+1)^2 & (\gamma+1)^2 \\ (\alpha-1)^2 & (\beta-1)^2 & (\gamma-1)^2 \end{vmatrix} = 0$, then

(a) all roots must be equal
(b) at least two roots are equal
(c) at least one root is non - zero
(d) none of these

5. Which of the following statements is (are) true?

(a) Every square matrix can be expressed as the sum of a symmetric and skew- symmetric matrices in unique way
(b) For a square matrix 'A', $|\text{Adj } A| = |A|^{n-1}$ where 'n' is the order of 'A'
(c) If 'A' is a non singular square matrix then $\left|A^{-1}\right| = \frac{1}{|A|}$
(d) Two matrices 'A' and 'B' are said to be conformable for multiplication in the same order iff the number of rows of 'A; is equal to number of columns of B

6. If for a square matrix $A = [a_{ij}]$, $a_{ij} = i^2 - j^2$ $i, j = 1, 2, \ldots, n$ is of even order then

(a) A is a skew-symmetric
(b) $|A|$ is not a perfect square
(c) A is symmetric and $|A| = 0$
(d) A is neither symmetric nor skew-symmetric

7. Let $S = \left\{(a, b) : A^3 = A, \text{where } A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ a & b \end{pmatrix}\right\}$ then $n(S) =$

(a) 1 (b) 2 (c) 3 (d) 0

8. If A and P are different matrices of order n satisfying $A^3 = P^3$ and $A^2P = P^2A$ (where $|A| \neq |P|$) then $|A^2 + P^2|$ is equal to

(a) n (b) 0 (c) $|A|\,|P|$ (d) $|A + P|$

9. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$ where ω is a complex cube root of unity then adj A equals

(a) $(\omega^2 - \omega)\bar{A}$ (b) $(\omega - \omega^2)\bar{A}$
(c) $-\bar{A}$ (d) $\bar{A}$

10. Let A and B be square matrices of same order satisfying $AB = A$ and $BA = B$, then A^2B^2 equals (O being zero matrices of the same order as B)

(a) A (b) B (c) I (d) O

11. The product of all the values of t, for which the system of the equations $(a - t)x + by + cz = 0$, $bx + (c - t)y + az = 0$, $cx + ay + (b - t)z = 0$ has non-trivial solution, is

(a) $\begin{vmatrix} a & -c & -b \\ -c & b & -a \\ -b & -a & c \end{vmatrix}$ (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

(c) $\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$ (d) none of these

12. The number of 2×2 matrices A, that are there with the elements as real numbers satisfying $A + A^T = I$ and $AA^T = I$ is

(a) zero (b) one
(c) two (d) infinite

13. If A, B, C, D be real matrices (not necessarily square) such that $A^T = BCD$, $B^T = CDA$, $C^T = DAB$, $D^T = ABC$ for the matrix $S = ABCD$ consider the statements

I : $S^3 = S$
II : $S^2 = S^4$

(a) II is true but not I
(b) I is true but not II
(c) both I and II are true
(d) both I and II are false

14. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value –1. Then

(a) C is empty
(b) B has as many elements as C
(c) $A = B \cup C$
(d) B has twice as many elements as C

15. If

$$\begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z}{r+2} \end{vmatrix} = \lambda \begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z}{r+2} \end{vmatrix}$$

then λ

(a) 0 (b) 1 (c) −1 (d) 2

16. If

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

then $f(100) =$

(a) 0 (b) 100 (c) 1 (d) −100

17. The value of 'a' for which the system of equations

$$\left.\begin{aligned} (a+1)^3 x + (a+2)^3 y &= (a+3)^3 \\ (a+1)x + (a+2)y &= a+3 \\ x + y &= 1 \end{aligned}\right\}$$ is consistent, is

(a) −2 (b) −1 (c) 1 (d) 2

18. Let A,B are square matrices of same order satisfying $AB = A$ and $BA = B$ then $(A^{2010} + B^{2010})^{2011}$ equals

(a) $A + B$ (b) $2010\ (A + B)$
(c) $2011\ (A + B)$ (d) $2^{2011}\ (A + B)$

19. The system of homogeneous equations $tx + (t + 1)y + (t - 1)z = 0$, $(t + 1)x + ty + (t + 2)z = 0$, $(t - 1)x + (t + 2)y + tz = 0$ has a non-trivial Solution for

(a) exactly three real values of t
(b) exactly two real values of t
(c) exactly one real value of t
(d) infinite number of values of t

20. If A is a square matrix of order 3 such that $|A| = 5$ then $|\text{adj}\ (4A)| =$

(a) $5^3 \times 4^2$ (b) $5^2 \times 4^3$
(c) $5^2 \times 16^3$ (d) $5^3 \times 16^2$

21. If p, q and r are in A.P. then the value of determinant

$$\begin{vmatrix} a^2 + 2^{n+1} + 2p & b^2 + 2^{n+2} + 3q & c^2 + p \\ 2^n + p & 2^{n+1} + q & 2q \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{vmatrix}$$ is

(a) 1 (b) 0
(c) $a^2b^2c^2 - 2^n$ (d) $(a^2 + b^2 + c^2) - 2^n q$

SECTION-II

More than One Correct Answer Type

22. Which of the following statements is always true?

(a) $A(\text{Adj}(A)) = (\text{Adj}(A))A$
(b) Adjoint of a diagonal matrix is a diagonal matrix
(c) Adjoint of a symmetric matrix is a symmetric matrix
(d) Adjoint of a unit matrix is a unit matrix

23. If

$$f(x) = \begin{vmatrix} 3 & 3x & 3x^2 + 2a^2 \\ 3x & 3x^2 + 2a^2 & 3x^3 + 6a^2x \\ 3x^2 + 2a^2 & 3x^3 + 6a^2x & 3x^4 + 12a^2x^2 + 2a^4 \end{vmatrix}$$

then

(a) $f'(x) = 0$
(b) $f(x)$ is independent of x
(c) $\int_0^1 f(x)dx = 4a^6$
(d) $y^2 = f(x)$ represents a pair of lines

24. If $A \begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 5 \\ -2 & 4 & -3 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 5 \\ 1 & 3 & 4 \\ +4 & -8 & 6 \end{pmatrix}$ then $A =$

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

25. If

$$\begin{vmatrix} x^2 - 5x + 3 & 2x - 5 & 3 \\ 3x^2 + x + 4 & 6x + 1 & 9 \\ 7x^2 - 6x + 9 & 14x - 6 & 21 \end{vmatrix} = ax^3 + bx^2 + cx + d$$

then which of the following are correct ?
(a) $a = 0$ (b) $b = 0$ (c) $c = 0$ (d) $d = 0$

26. If $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ which of the following are correct?

(a) $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \Delta^2$

(b) $\begin{vmatrix} a^2 & c^2 & 2ac-b^2 \\ 2ab-c^2 & b^2 & a^2 \\ b^2 & 2bc-a^2 & c^2 \end{vmatrix} = \Delta^2$

(c) $\Delta = 0 \Rightarrow$ either $a = b = c$ or $a + b + c = 0$
(d) $\Delta = 3abc - a^3 - b^3 - c^3$

27. If $f(x)$ and $g(x)$ are functions such that $f(x + y) = f(x) \cdot g(y) + g(x) \cdot f(y)$, then

$\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha+\theta) \\ f(\beta) & g(\beta) & f(\beta+\theta) \\ f(\gamma) & g(\gamma) & f(\gamma+\theta) \end{vmatrix}$ is independent of

(a) α (b) β (c) γ (d) θ

28. If $AB = A$ and $BA = B$, then
(a) $B = I$ (b) $A = I$
(c) $A^2 = A$ (d) $B^2 = B$

29. If the equations $x + ay - z = 0$, $2x - y + az = 0$, $ax + y + 2z = 0$ have non-trivial solutions, then $a =$
(a) 2 (b) -2 (c) $1+\sqrt{3}$ (d) $1-\sqrt{3}$

30. If $A^2 + A + I = 0$, then
(a) A is non-singular (b) $A \neq 0$
(c) A is singular (d) $A^{-1} = -(A + I)$

31. If the elements of a 2×2 matrix A are real positive and distinct such that $\det(A + A^T)^T = 0$ then
(a) $\det A > 0$ (b) $\det A \geq 0$
(c) $\det(A - A^T) > 0$ (d) $\det(AA^T) > 0$

32. Let $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$ and if AB is a scalar multiple of B, then

(a) $4a + 7b + 5 = 0$ (b) $a + b + 2 = 0$
(c) $b - a = 4$ (d) $a + 3b = 0$

33. If 'A' is an odd order sequence matrix such that $A^2 + A + 2I = 0$ then which of the following is/are true ?
(a) A is non-singular
(b) A is symmetric
(c) A cannot be skew symmetric
(d) $A^{-1} = -\frac{1}{2}(A + I)$

34. $A \in M_n(c)$ (where M is a matrix of order $n \times n$) is non-singular matrix such that $3ABA^{-1} + A = 2A^{-1}BA$ then
(a) A & B both are identity matrices
(b) $|A + B| = 0$
(c) $|ABA^{-1} - AB^{-1}A| = 0$
(d) $A + B$ is not a singular matrix

35. A and B are square matrices with the same order. $AB = BA$ and A and B are skew hermitian.
(a) $A + B$ is skew hermitian
(b) AB is skew hermitian
(c) The diagonal elements of A and B are all zero.
(d) AB is hermitian

36. Let a, b, c be real numbers such that $a^2 + b^2 + c^2 = 1$ then $\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$ is a
(a) Straight line (b) Point circle
(c) Parabola (d) Ellipse

37. If $M = \{A : A$ is 3×3 matrix whose entries are -1 and $1\}$ then
(a) $\det A$ lies between -6 and 6
(b) $\det A \in \{-4, 0, 4\}$
(c) Number of elements of $M = 2^9$
(d) Number of elements of $M = 3^9$

38. Let

$$\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

then
(a) $f = 1$ (b) $e = 3$
(c) $a + c = -1$ (d) $b + d = 1$

SECTION-III

Comprehension Type

Paragraph for Question No. 39 to 41

A square matrix A is said to be orthogonal if $A^TA = AA^T = I$ with this information answer the following.

39. $P_{2\times 2}$ is orthogonal matrix, $A = \begin{bmatrix} 29 & -28 \\ 30 & -29 \end{bmatrix}$. If $Q = P^T AP$. Then $PQ^{2012}P^T =$

(a) $2012\,A$ (b) I (c) A (d) A^{2011}

40. $P_{3\times 3}$ orthogonal matrix, α, β, γ are the angles made by a straight line with OX, OY, OZ.

$$A = \begin{bmatrix} \sin^2\alpha & \sin\alpha\sin\beta & \sin\alpha\sin\gamma \\ \sin\alpha\sin\beta & \sin^2\beta & \sin\beta\sin\gamma \\ \sin\alpha\sin\gamma & \sin\beta\sin\gamma & \sin^2\gamma \end{bmatrix}$$

and $Q = P^T$ AP. If $PQ^6P^T = 2^kA$, then $k =$

(a) 5 (b) 7 (c) 6 (d) 0

41. $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal matrix then

$36\,|abc| =$

(a) 4 (b) 6 (c) 9 (d) 1

Paragraph for Question No. 42 to 44

If A and B are square matrices then

42. If $AB + BA = O$ then which of the following option is equivalent to $A^3 - B^3$?

(a) $(A - B)(A^2 + AB + B^2)$

(b) $(A - B)(A^2 - AB - B^2)$

(c) $(A + B)(A^2 - AB - B^2)$

(d) $(A + B)(A^2 + AB - B^2)$

43. If A, B are non-singular matrices such that, $B \neq I$, $A^6 = I$ & $AB^2 = BA$. Then the least value of k for $B^k = I$ is

(a) 7 (b) 15 (c) 63 (d) 127

44. $|A - B| \neq 0$, $A^4 = B^4$, $C^3A = C^3B$, $B^3A = A^3B$ then $|A^3 + B^3 + C^3| =$

(a) 0 (b) 1 (c) $3|A|^3$ (d) 6

Paragraph for Question No. 45 to 47

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, U_1, U_2, U_3 are column matrices satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is 3×3 matrix whose columns are U_1, U_2, U_3

45. The value of det U is

(a) 3 (b) -3 (c) $\frac{3}{2}$ (d) 2

46. The sum of the elements of U^{-1} is

(a) -1 (b) 0 (c) 1 (d) 3

47. $\begin{bmatrix} 3 & 2 & 0 \end{bmatrix} U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} =$

(a) 5 (b) 5/2 (c) 4 (d) 3/2

Paragraph for Question No. 48 to 50

A square matrix A is said to be unitary matrix iff $AA^* = I = A^*A$ where A^* is the conjugate transpose of A. Given that $A = k\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ where $k \in R$

48. If A is an unitary matrix, then k is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$

(c) Both (a) and (b) (d) Either (a) or (b)

49. If $Q = ABA^*$ where $B = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then the value of $X = A^*Q^{20}A$ is

(a) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

(b) $\begin{bmatrix} \cos 20\alpha & \sin 20\alpha \\ -\sin 20\alpha & \cos 20\alpha \end{bmatrix}$

(c) $\begin{bmatrix} \cos 20\alpha & -\sin 20\alpha \\ \sin 20\alpha & \cos 20\alpha \end{bmatrix}$

(d) $\begin{bmatrix} \cos 20\alpha & -\sin 20\alpha \\ -\sin 20\alpha & \cos 20\alpha \end{bmatrix}$

50. The value of $|\text{adj}\,(\text{adj}X)|$ is

(a) 1 (b) 0 (c) -1 (d) 2

SECTION-IV

Matrix-Match Type

51. Given system of equations $x + y + 2z = a$, $-2x - z = b$, $x + 3y + 5z = c$, (a, b, c) is an ordered triplet then

Column-I		Column-II	
(A)	(0, 0, 0)	(p)	no solution
(B)	(−1, 1, 1)	(q)	infinite solution
(C)	(1, 1, 4)	(r)	non-trivial solution
(D)	(2, 1, −1)	(s)	trivial solution

52. Suppose matrix $A=\begin{bmatrix}7 & a & b & 1\\ c & \alpha & \beta & d\\ 3 & e & f & 10\end{bmatrix}$. All the unknown numbers are distinct integers from set {2, 4, 5, 6, 8, 9} such that sum of the entries in 1st row, 3rd row, 1st column and 4th column are equal to k then

Column-I		Column-II	
(A)	k =	(p)	20
(B)	$a+b+c$ =	(q)	17
(C)	$2ef$	(r)	19
(D)	$c+d$	(s)	16

53. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and $3B = A$ then match column-I with column-II.

Column-I		Column-II	
(A)	adj A is	(p)	Orthogonal matrix
(B)	B is	(q)	Involutory matrix
(C)	B^T is	(r)	Cofactor matrix of A
(D)	B^{-1}	(s)	$-3A$

54. If

$$p(\theta)=\begin{vmatrix}-\sqrt{2} & \sin\theta & \cos\theta\\ 1 & \cos\theta & \sin\theta\\ -1 & \sin\theta & -\cos\theta\end{vmatrix}, q(\theta)=\begin{vmatrix}\sin 2\theta & -1 & 1\\ \cos 2\theta & 4 & -3\\ 2 & 7 & -5\end{vmatrix}$$

$$r(\theta)=\begin{vmatrix}\cos\theta & \sin\theta & \cos\theta\\ -\sin\theta & \cos\theta & \sin\theta\\ -\cos\theta & -\sin\theta & \cos\theta\end{vmatrix},$$

$$s(\theta)=\begin{vmatrix}\sec^2\theta & 1 & 1\\ \cos^2\theta & \cos^2\theta & \operatorname{cosec}^2\theta\\ 1 & \cos^2\theta & \cot^2\theta\end{vmatrix}$$

Then match the following.

Column-I Function		Column-II Range	
(A)	$p(\theta)$	(p)	$[0, 1]$
(B)	$q(\theta)$	(q)	$[0, 2\sqrt{2}]$
(C)	$r(\theta)$	(r)	$[-2, 2]$
(D)	$s(\theta)$	(s)	$[-\sqrt{5}-2, \sqrt{5}-2]$

55.

Column-I		Column-II	
(A)	If ω is a non real cube root of unity, then a root of the following equation $\begin{vmatrix}x+1 & \omega & \omega^2\\ \omega & x+\omega^2 & 1\\ \omega^2 & 1 & x+\omega\end{vmatrix}=0$ is	(p)	0
(B)	If $i^2 = -1$, ω is a non-real cube root of unity and if $a = i$, $b = \omega$ and $c = \omega^2$, then the value of determinant $\begin{vmatrix}a & a+b & a+b+c\\ 3a & 4a+3b & 5a+4b+3c\\ 6a & 9a+6b & 11a+9b+6c\end{vmatrix}$	(q)	2
(C)	If ω is a non-real cube root of unity, the value of determinant $\begin{vmatrix}1 & 1 & 1\\ 1 & \omega^2 & \omega\\ 1 & \omega & \omega^2\end{vmatrix}$ is equal to	(r)	i
(D)	Let $\begin{vmatrix}x & 2 & x\\ x^2 & x & 6\\ x & x & 6\end{vmatrix}$ $= Ax^4 + Bx^3 + Cx^2 + Dx + E$ then the value of $A + B + C + D + E$ =	(s)	$3\sqrt{3}i$

SECTION-V

Integer Answer Type

56. If $x - 2y + z = -4$, $2x - y + 2z = 2$, $x + y + \lambda z = 4$ are inconsistent then λ =

57. If $\Delta_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} \Delta_k = 56$, then n is equal to

58. If $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$, $\left(-\frac{\pi}{4} \le x \le \frac{\pi}{4}\right)$ then $\tan x =$

59. If x, y, z are non-zero real numbers and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$ then $-\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) =$

60. If $\begin{bmatrix} 12 & \alpha^2 \\ -16 & -12 \end{bmatrix}$ is a nilpotent matrix of index 2, $\alpha > 0$ then $\alpha =$

61. If $f(\theta) = \begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix}$ then number of solutions of $f(\theta) = 0$ in $\left[0, \frac{\pi}{2}\right]$ is

62. Rank of $\begin{bmatrix} 2 & 4 & 7 \\ 17 & 38 & 57 \\ 33 & 70 & 113 \end{bmatrix}$ is

63. If $f(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\beta) & \sin(\alpha-\gamma) \end{vmatrix}$ and $f'(2) = \frac{1}{4}$ then $\left[\sum_{r=1}^{15} f(x_r)\right]$ is (where $[\cdot]$ denotes greatest integer function)

64. If the system of equations $3x - 2y + z = 0$, $\lambda x - 14y + 15z = 0$, $x + 2y - 3z = 0$ has a non-zero solution, then $\lambda =$

SOLUTIONS

1. (a) : $\sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} r & x & \frac{n(n+1)}{2} \\ \sum_{r=1}^{n} (2r-1) & y & n^2 \\ \sum_{r=1}^{n} (3r-2) & z & \frac{n(3n-1)}{2} \end{vmatrix}$

$= \begin{vmatrix} \frac{n(n+1)}{2} & x & \frac{n(n+1)}{2} \\ n^2 & y & n^2 \\ \frac{n(3n-1)}{2} & z & \frac{n(3n-1)}{2} \end{vmatrix} = 0 \quad (\because C_1 \approx C_2)$

2. (c) :

$[AD] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{array}\right] R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$[AD] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{array}\right]$

It has no solution for $a = 3$, $b \neq 10$

3. (b) : By Cayley Hamilton theorem, characteristic equation of A is given by $|A - \lambda I| = 0$

$\Rightarrow \lambda^2 - 5\lambda - 2I = 0$

$\Rightarrow A^2 - 5A - 2I = 0$

Now $A^4 - 5A^3 - A^2 - 4A - 2I$

$= A^2(A^2 - 5A - 2I) + A^2 - 4A - 2I$

$= A^2(0) + A^2 - 5A - 2I + A = A$

4. (a) : $R_2 \to R_2 - R_3$

$\begin{vmatrix} \alpha^2 & \beta^2 & \gamma^2 \\ \alpha & \beta & \gamma \\ (\alpha-1)^2 & (\beta-1)^2 & (\gamma-1)^2 \end{vmatrix} = 0$

Apply $R_3 \to R_3 - R_1$, we get

$\begin{vmatrix} \alpha^2 & \beta^2 & \gamma^2 \\ \alpha & \beta & \gamma \\ 1-2\alpha & 1-2\beta & 1-2\gamma \end{vmatrix} = 0$

Applying $R_3 \to R_3 + 2R_2$, we get

$$\begin{vmatrix}\alpha^2 & \beta^2 & \gamma^2\\ \alpha & \beta & \gamma\\ 1 & 1 & 1\end{vmatrix}=0\Rightarrow(\beta-\alpha)(\gamma-\alpha)(\beta-\gamma)=0$$

$$\Rightarrow \alpha=\beta=\gamma$$

5. (d) : (a) $A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)$

where $\frac{1}{2}(A+A^T)$ is symmetric and $\frac{1}{2}(A-A^T)$ is skew-symmetric

(b) $A(\text{Adj } A)=|A|I$

$\Rightarrow |A||\text{Adj } A|=|A|^n\Rightarrow|\text{Adj } A|=|A|^{n-1}$

(c) $A^{-1}=\frac{\text{Adj}A}{|A|}\Rightarrow\left|A^{-1}\right|=\frac{1}{|A|^n}|A|^{n-1}=\frac{1}{|A|}$

(d) product AB is possible when number of columns of 'A' is equal to number of rows of 'B'

6. (a) : $A=\begin{bmatrix}0 & 1^2-2^2 & 1^2-3^2....\\ 2^2-1^2 & 0 & \\ 3^2-1^2 & & 0\\ \vdots & & \end{bmatrix}$

$\therefore$ A is skew-symmetric of even order

$\therefore$ $|A|$ = perfect square

7. (c) : Characteristic polynomial of A is

$$|A-\lambda I|=\begin{vmatrix}\frac{1}{2}-\lambda & \frac{1}{2}\\ a & b-\lambda\end{vmatrix}=0=\lambda^2-p\lambda+q$$

Where $p=b+\frac{1}{2},\ q=\frac{b-a}{2}$ .....(1)

According to cayley hamilton theorem

$A^2-pA+qI=0\ \ \therefore\ A^2=pA-qI$

$\Rightarrow A^3=pA^2-qA=p(pA-qI)-qA$

$A^3=A\Rightarrow(p^2-q-1)A=pqI$

$\Rightarrow p^2-q-1=pq=0\Rightarrow q=p^2-1\ \&\ p(p^2-1)=0$

(as $pq=0\Rightarrow p(p^2-1)=0$)

$\Rightarrow (p,q)=(-1,0),(0,-1),(1,0)$

Correspondingly we get

$(a,b)=\left(\frac{-3}{2},\frac{-3}{2}\right),\left(\frac{3}{2},\frac{-1}{2}\right),\left(\frac{1}{2},\frac{1}{2}\right)$

By using (1) $p=0$, $q=-1$ in (1)

$a=\frac{3}{2},\ b=-\frac{1}{2}$

(ii) $p=-1$, $q=0$ we get $a=b=-3/2$

(iii) $p=1$, $q=0$ we get $a=b=1/2$

8. (b) : Since $A\neq P$, we have

$(A^2+P^2)(A-P)=A^3-A^2P+P^2A-P^3=O_n$

Hence A^2+P^2 has zero divisor.

9. (b) :

$$A=\begin{pmatrix}1 & 1 & 1\\ 1 & \omega^2 & \omega\\ 1 & \omega & \omega^2\end{pmatrix}\quad\therefore\ \bar{A}=\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{pmatrix}$$

Cofactors of elements

1, 1, 1 are $\omega-\omega^2,\ \omega-\omega^2,\ \omega-\omega^2$

1, ω^2, ω are $\omega-\omega^2,\ \omega^2-1,\ 1-\omega$

1, ω, ω^2 are $\omega-\omega^2,\ 1-\omega,\ \omega^2-1$

Note

1. $1-\omega=\omega^3-\omega^4=\omega^2(\omega-\omega^2)$ and $\omega^2-1=\omega^2-\omega^3=\omega(\omega-\omega^2)$
2. Conjugate of ω is ω^2 and conjugate of ω^2 is ω.

$\therefore$ Matrix of cofactors is

$$\begin{pmatrix}\omega-\omega^2 & \omega-\omega^2 & \omega-\omega^2\\ \omega-\omega^2 & \omega(\omega-\omega^2) & \omega^2(\omega-\omega^2)\\ \omega-\omega^2 & \omega^2(\omega-\omega^2) & \omega(\omega-\omega^2)\end{pmatrix}$$

$$=(\omega-\omega^2)\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{pmatrix}=(\omega-\omega^2)\bar{A}$$

10. (a) : $AB=A,\ BA=B$

$\Rightarrow (AB)B=(A)B=A$

$\Rightarrow A(AB^2)=A\cdot A=A^2=A^2B^2$ .....(1)

$A(BA)=A(B)$

$\Rightarrow A^2=A$ .....(2)

From (1) and (2), $A^2B^2=A$.

11. (b) : For the given system of equations to have non-trivial solutions $\begin{vmatrix}a-t & b & c\\ b & c-t & a\\ c & a & b-t\end{vmatrix}=0$ which is a cubic in t with coefficient of t^3 as unity and the constant term as $-\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$.

Hence product of all the values of $t=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$.

12. (c) : $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$A+A^T=I \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}+\begin{pmatrix} a & c \\ b & d \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\Rightarrow \quad 2a=1, b+c=0, 2d=1$

$\Rightarrow \quad a=\frac{1}{2}, c=-b, \; d=\frac{1}{2}$

So, $A=\begin{pmatrix} 1/2 & b \\ -b & 1/2 \end{pmatrix}$

Now $AA^T=I \Rightarrow \begin{pmatrix} 1/2 & b \\ -b & 1/2 \end{pmatrix}\begin{pmatrix} 1/2 & -b \\ b & 1/2 \end{pmatrix}=I$

$$\Rightarrow \begin{pmatrix} \frac{1}{4}+b^2 & 0 \\ 0 & b^2+\frac{1}{4} \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad b^2+\frac{1}{4}=1 \Rightarrow b=\pm\frac{\sqrt{3}}{2}$$

$$\therefore \quad A=\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \& \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

Number of matrices = 2

13. (c) : $S^3 = (ABCD)(ABCD)(ABCD)$

$= (ABC)(DAB)(CDA)(BCD)$

$S^3 = D^TC^TB^TA^T = D^T(ABC)^T = D^TD = ABCD = S$

$\therefore \quad S^3 = S \Rightarrow S^4 = S^2$

14. (b) : If we interchange any two rows of a determinant in the set B, its value becomes -1 and hence it is in C. Likewise, for every determinant in C, there is a corresponding determinant in B. $\therefore$ B and C have the same number of elements.

15. (b) : We use the pascal rule

$$\binom{n}{r}+\binom{n}{r+1}=\binom{n+1}{r+1} \text{ or } \binom{n+1}{r+1}-\binom{n}{r}=\binom{n}{r+1}$$

repeatedly in transforming the determinant

$$\begin{vmatrix} \binom{x}{r} & \binom{x+1}{r+1} & \binom{x+2}{r+2} \\ \binom{y}{r} & \binom{y+1}{r+1} & \binom{y+2}{r+2} \\ \binom{z}{r} & \binom{z+1}{r+1} & \binom{z+2}{r+2} \end{vmatrix} \begin{matrix} \\ \\ C_3-C_2 \to C_3 \\ C_2-C_1 \to C_2 \end{matrix}$$

$$= \begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x+1}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y+1}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z+1}{r+2} \end{vmatrix} C_3-C_2 \to C_3$$

$$= \begin{vmatrix} \binom{x}{r} & \binom{x}{r+1} & \binom{x}{r+2} \\ \binom{y}{r} & \binom{y}{r+1} & \binom{y}{r+2} \\ \binom{z}{r} & \binom{z}{r+1} & \binom{z}{r+2} \end{vmatrix} \therefore \lambda = 1$$

16. (a) : Taking common factors x from C_2,$(x + 1)$ from C_3 and $(x - 1)$ from R_3, we have

$$f(x)=x(x^2-1)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix} \begin{matrix} \\ C_3-C_2 \to C_3 \\ C_2-C_1 \to C_2 \end{matrix}$$

$$= x(x^2-1)\begin{vmatrix} 1 & 0 & 0 \\ 2x & -(x+1) & 1 \\ 3x & -2(x+1) & 2 \end{vmatrix} = 0$$

$$f(100)=100\times 9999\begin{vmatrix} 1 & 0 & 0 \\ 200 & -101 & 1 \\ 300 & -202 & 2 \end{vmatrix}$$

$\therefore \quad f(100) = 0$

17. (a) : The system of equations is consistent if the determinant of coefficients is zero

$$(i.e.) \begin{vmatrix} (a+1)^3 & (a+2)^3 & (a+3)^3 \\ a+1 & a+2 & a+3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$(i.e.) \begin{vmatrix} (a+1)^3 & 3a^2+9a+7 & 3a^2+15a+19 \\ a+1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

By $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$

$(i.e.)$ $(3a^2 + 9a + 7) - (3a^2 + 15a + 19) = 0$

$(i.e.)$ $-6a - 12 = 0$ or $a = -2$

18. (d) : $\left.\begin{matrix} AB=A \\ BA=B \end{matrix}\right\} \Rightarrow \left.\begin{matrix} A^2=A \\ B^2=B \end{matrix}\right\} \Rightarrow \left.\begin{matrix} A^n=A \\ B^n=B \end{matrix}\right\}$

$\because$ $AB = A$ and $BA = B$

$ABA = A^2$ $B(AB) = B^2$

$AB = A^2$ $BA = B^2$

$A = A^2$ $B = B^2$

$A^2 = A^3 = A$ Similarly

$A^{2010} = A$ $B^{2010} = B$

$(A + B)^2 = A^2 + B^2 + AB + BA = 2(A + B)$

.....

.....

$(A + B)^k = 2^k (A + B)$

$\therefore$ $(A^{2010} + B^{2010})^{2011} = (A + B)^{2011} = 2^{2011}(A + B)$

19. (c) : To have a non-trivial solution

$$\begin{vmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{vmatrix} = 0 \Rightarrow 2t+1=0 \Rightarrow t=-\frac{1}{2}$$

20. (c) : Adj $(4A) = 4^2$ Adj$(A) = 16$ Adj(A)

$\Rightarrow$ $|\text{Adj } 4A| = 16^3$. $|\text{Adj } A| = 16^3 \cdot 5^2$

21. (b) :
$$\begin{vmatrix} a^2+2^{n+1}+2p & b^2+2^{n+2}+3q & c^2+p \\ 2^n+p & 2^{n+1}+q & 2q \\ a^2+2^n+p & b^2+2^{n+1}+2q & c^2-r \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$ and $2q = p + r$, we get

$$\begin{vmatrix} 2^{n+1}-2^n+p & 2^{n+2}-2^{n+1}+q & p+r \\ 2^n+p & 2^{n+1}+q & 2q \\ a^2+2^n+p & b^2+2^{n+1}+2q & c^2-r \end{vmatrix}$$

$$\begin{vmatrix} 2^n(2-1)+p & 2^{n+1}(2-1)+q & 2q \\ 2^n+p & 2^{n+1}+q & 2q \\ a^2+2^n+p & b^2+2^{n+1}+2q & c^2-r \end{vmatrix} = 0 \ (R_1 \approx R_2)$$

$\because$ $2q = p + r$

22. (a, b, c, d) : All are standard results

23. (a, b, c, d) : Applying $C_3 \to C_3 - xC_2$, $C_2 \to C_2 - xC_1$

$$f(x) = \begin{vmatrix} 3 & 0 & 2a^2 \\ 3x & 2a^2 & 4a^2x \\ 3x^2+2a^2 & 4a^2x & 6a^2x^2+2a^4 \end{vmatrix}$$

$$= 4a^4 \begin{vmatrix} 3 & 0 & 1 \\ 3x & 1 & 2x \\ 3x^2+2a^2 & 2x & 3x^2+a^2 \end{vmatrix}$$

Applying $C_3 \to C_3 - xC_2$, we get

$$= 4a^4 \begin{vmatrix} 3 & 0 & 1 \\ 3x & 1 & x \\ 3x^2+2a^2 & 2x & x^2+a^2 \end{vmatrix}$$

Applying $C_1 \to C_1 - 3C_3$, we get

$$= 4a^4 \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & x \\ -a^2 & 2x & x^2+a^2 \end{vmatrix} = 4a^6$$

24. (d) : By observation its obvious that R_1 & R_2 are interchanged & R_3 of R.H.S – 2 times R_3 of L.H.S

i.e. $R_1 \to R_2$, $R_3 \to -2R_3$ $\Rightarrow$ $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

25. (a, b, c) :

$$f'(x) = \begin{vmatrix} 2x-5 & 2x-5 & 3 \\ 6x+1 & 6x+1 & 9 \\ 14x-6 & 14x-6 & 21 \end{vmatrix} + \begin{vmatrix} x^2-5x+3 & 2 & 3 \\ 3x^2+x+4 & 6 & 9 \\ 7x^2-6x+9 & 14 & 21 \end{vmatrix} = 0+0=0$$

$\Rightarrow$ $f(x)$ is a constant polynomial & $f(0) \neq 0$

$\Rightarrow$ $d \neq 0$

26. (a, b, c, d) : Replace every element of Δ by its cofactors we get $(A) = \Delta^2$.

Also $\Delta^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \begin{vmatrix} a & b & c \\ -c-a & -b & \\ b & c & a \end{vmatrix} = (b)$

Here $\Delta = 3abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow$ either $a = b = c$ or $a + b + c = 0$.

27. (a, b, c, d) : Expand third column and express it as sum of two determinants then two columns become identical. So, determinant value becomes equal to zero.

28. (c, d) : $A = AB = A(BA) = (AB)\,A = A \cdot A = A^2$
$B = BA = B(AB) = (BA)B = B \cdot B = B^2$

29. (b, c, d) : $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 \end{vmatrix} = 0 \quad \begin{matrix} R_2 - 2R_1 \to R_2 \\ R_3 - aR_1 \to R_3 \end{matrix}$

$$\begin{vmatrix} 1 & a & 1 \\ 0 & -1-2a & a+2 \\ 0 & 1-a^2 & 2+a \end{vmatrix} = 0 \Rightarrow (a+2)(a^2-2a-2) = 0$$

$\Rightarrow \quad a = -2,\ 1 \pm \sqrt{3}$

30. (a, b, d) : $A^2 + A + I \Rightarrow A(A + I) = -I$
$\Rightarrow \quad A \neq 0.\ \det A \det(A + I) = (-1)^n$
$\Rightarrow \quad \det A \neq 0$
Also $A^{-1} = -(A + I)$

31. (a, c, d) : $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad a \neq b \neq c \neq d > 0$

$$A + A^T = \begin{pmatrix} 2a & b+c \\ b+c & 2d \end{pmatrix}$$

$$\left|A + A^T\right| = 4ad - (b+c)^2 = 0 \Rightarrow b + c = 2\sqrt{ad}$$

$\Rightarrow \quad b + c = 2\sqrt{ad} > 2\sqrt{bc}$ (A.M. > G.M.)
$\Rightarrow \quad ad > bc \qquad b \neq c$
$\Rightarrow \quad ad - bc > 0$
$\Rightarrow \quad \det A > 0$

$$\left|A - A^T\right| = \begin{vmatrix} 0 & b-c \\ c-b & 0 \end{vmatrix} = 0 + (b-c)^2 > 0$$

$|AA^T| = |A|\,|A^T| = |A|^2 = (\det A)^2 > 0$

32. (a, b, c, d) : $AB = \lambda B \Rightarrow (A - \lambda I).B = 0$

$$\Rightarrow \begin{bmatrix} -3-\lambda & -7 & -5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} a \\ b \\ 1 \end{bmatrix} = 0$$

$\Rightarrow \quad \det(A - \lambda I) = 0 \Rightarrow (\lambda - 1)^3 = 0 \Rightarrow \lambda = 1$
$\therefore \quad 4a + 7b + 5 = 0,\ 2a + 3b + 3 = 0$
$\Rightarrow \quad a = -3,\ b = 1,\ a + b = -2$
$4a + 7b + 5 = 0,\ a + b + 2 = 0,\ b - a = 4$ and
$a + 3b = 0$

33. (a,c,d) : $A^2 + A + 2I = 0$
$A^2 + A = -2I$
$|A^2 + A| = |-2I|$
$|A|\,|A + I| = (-2)^n$
$|A| \neq 0$
'A' is non-singular. Hence its inverse exists. Also multiplying the given equation both sides with A^{-1}

we get $A^{-1} = -\frac{1}{2}(A + I)$

34. (b, c) : $A \in M_n(C)$ (Where M matrix of order $n \times n$) is non-singular matrix such that
$3ABA^{-1} + A = 2A^{-1}BA$
$\Rightarrow\ 3ABA^{-1} + A + 2A = 2A^{-1}\,BA + 2A$
$\Rightarrow \quad 3A(BA^{-1} + I) = 2\,(A^{-1}\,B + I)A$
$\Rightarrow \quad 3A(B + IA)A^{-1} = 2A^{-1}(B + AI)A.$
Let $B + AI = X$
$\Rightarrow \quad 3AXA^{-1} = 2A^{-1}XA$
$\Rightarrow \quad 3^n|A||X||A^{-1}| = 2^n|A^{-1}||X||A| \Rightarrow 3^n|X| = 2^n|X|$
(Cancellation is allowed because A is non-singular)
$3^n|X| = 2^n|X| \Rightarrow |X| = 0$ *i.e* $|A + B| = 0$ .....(1)
Let $M = ABA^{-1} - A^{-1}BA$
$AM = A^2BA^{-1} - BA \Rightarrow BA = A^2BA^{-1} - AM$
$3ABA^{-1} + A = 2A^{-1}\,BA = 2A^{-1}(A^2BA^{-1} - AM)$
$\qquad\qquad = 2ABA^{-1} - 2M$
$\Rightarrow \quad ABA^{-1} + A = -2M \Rightarrow A(BA^{-1} + I) = -2M$
Taking determinants both sides, we get
$|-2M| = |A||A + B||A^{-1}| = 0$ (from (1))
$\Rightarrow \quad |ABA^{-1} - AB^{-1}A| = 0$

35. (a, c, d) : As A and B are skew hermitian
$\therefore \quad A^\theta = -A,\ B^\theta = -B$
$(A + B)^\theta = A^\theta + B^\theta$
$\qquad = -A - B = -(A + B)$ (a is true)
$(AB)^\theta = B^\theta\,A^\theta = (-B)(-A) = BA$
$\qquad = AB$ (b is false and d is true)
(c) is well-known as diagonal elements in hermitian matrix must be real.

36. (a) :

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2x - aby - ac & abx + a^2y & acx + a^2 \\ b^2x + aby & -abx + b^2y - bc & bcy + b^2 \\ c^2x + ca & c^2y + bc & -acx - bcy + c^2 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$= \frac{1}{abc}\begin{vmatrix} x & y & 1 \\ b^2x + aby & -abx + b^2y - bc & bcy + b^2 \\ c^2x + ca & c^2y + bc & -acx - bcy + c^2 \end{vmatrix}$$

$$= \frac{1}{a}\begin{vmatrix} x & y & 1 \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix}$$

Applying $R_2 \to R_2 - bR_1$, $R_3 \to R_3 - cR_1$, we get

$$= \frac{1}{a}\begin{vmatrix} x & y & 1 \\ ay & -ax - c & cy \\ a & b & -ax - by \end{vmatrix}$$

Applying $C_3 \to C_3 + xC_1 + yC_2$, we get

$$= \frac{1}{a}\begin{vmatrix} x & y & 1 + x^2 + y^2 \\ ay & -ax - c & 0 \\ a & b & 0 \end{vmatrix}$$

$$= \frac{1}{a}(x^2 + y^2 + 1)(ax + aby + ac)$$

$$= (x^2 + y^2 + 1)(ax + by + c)$$

$$\Rightarrow \quad (x^2 + y^2 + 1)(ax + by + c) = 0$$

37. (a, b, c) : $M = \{A/A$ is 3×3 matrix whose entries are -1 and $1\}$

Det $A = P_1 + P_2 + P_3 - P_4 - P_5 - P_6$ where $|P_i| = 1$

For example

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 4 & 1 \end{vmatrix} = -5.$$ Evaluation process $\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 1 & 3 & 4 & 1 & 3 \\ 2 & 4 & 1 & 2 & 4 \end{matrix}$

$$P_1 + P_2 + P_3 - P_4 - P_5 - P_6 = 3 + 16 + 12 - 18 - 16 - 2 = -5$$

Since each element of a matrix are either -1 or 1. So, product of any three numbers is -1 but modulus of all p_i are equal to 1.

$$|\det A| \le \sum_{i=1}^{3} |p_i| = 6 \Rightarrow -6 \le \det A \le 6.$$

Hence first option is correct.

Applying $C_2 \to C_2 + C_3$ and $C_3 \to C_3 + C_1$ we get second and third columns whose entries are even numbers.So $\det A = 4 \det B$, where B is a matrix with integer entries.

Hence 4 divides $\det A \Rightarrow \det A \in \{-4, 0, 4\}$.

Every element from A can be chosen from either '-1' or '1'. Hence number of possible matrices are 2^9.

38. (a, b, c, d) :

Let $f(x) = \begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix}$

$= ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

$\therefore \quad f(0) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = f \quad \therefore \ f = 1$

or $f(x) = (1+x)[(1+x)^2 - x^3] - x(x + x^2 - x^4) + x^4(x^2 - x^2 - x^3)$

$= (1+x)^3 - x^3 - x^4 - x^2 - x^3 + x^5 - x^5$

$= 0x^5 - 1x^4 - 1x^3 + 2x^2 + 3x + 1$

$= ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

$\therefore \ f = 1, e = 3, a = 0, b = -1, c = -1, d = 2$

(a) $f = 1$, (b) $e = 3$ (c) $a + c = -1$ (d) $b + d = 1$

39. (b) : $PQP^T = PP^T APP^T = IAI = A$

$PQ^2P^T = PQ \cdot QP^T = P.P^T AP\ P^TAPP^T = IAIAI = A^2$

and so on $PQ^{2012}P^T = A^{2012}$

But $A^2 = \begin{bmatrix} 29 & -28 \\ 30 & -29 \end{bmatrix}\begin{bmatrix} 29 & -28 \\ 30 & -29 \end{bmatrix}$

$$= \begin{bmatrix} 841 - 40 & -29 \times 28 + 28 \times 29 \\ 30 \times 29 - 29 \times 30 & -840 + 841 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\Rightarrow \quad A^{2012} = I$

40. (a) : $PQ^6P^T = A^6$.

$$A^2 = \begin{bmatrix} \sin^2\alpha & \sin\alpha\sin\beta & \sin\alpha\sin\gamma \\ \sin\alpha\sin\beta & \sin^2\beta & \sin\beta\sin\gamma \\ \sin\alpha\sin\gamma & \sin\beta\sin\gamma & \sin^2\gamma \end{bmatrix} \times$$

$$\begin{bmatrix} \sin^2\alpha & \sin\alpha\sin\beta & -\sin\alpha\sin\gamma \\ \sin\alpha\sin\gamma & \sin^2\beta & \sin\beta\sin\gamma \\ \sin\alpha\sin\gamma & \sin\beta\sin\gamma & \sin^2\gamma \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\alpha(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) & \sin\alpha\sin\beta(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) & \sin\alpha\sin\gamma(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) \\ \sin\alpha\sin\beta(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) & \sin^2\beta(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) & \sin\alpha\sin\gamma(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) \\ \sin\alpha\sin\gamma(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) & \sin\beta\sin\gamma(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) & \sin^2\gamma(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) \end{bmatrix}$$

$(\because l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1)$

$\therefore \quad A^2 = 2A \ (\because \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2)$

$\Rightarrow \quad A^3 = A \cdot A^2 = A(2A) = 2(2A) = 2^2A$

$\Rightarrow \quad A^6 = 2^5A$

41. (b) : A is orthogonal matrix $\Rightarrow AA^T = I$

$\Rightarrow \quad 2a^2 = 1, 6b^2 = 1, 3c^2 = 1 \Rightarrow a^2b^2c^2 = \dfrac{1}{36}$

$\Rightarrow \quad |abc| = \dfrac{1}{6} \Rightarrow 36|abc| = 6$

42. (c) : $(A + B)(A^2 - AB - B^2)$

$= A^3 - A^2B - AB^2 + BA^2 - BAB - B^3$

$= A^3 - B^3 - A^2B - AB^2 - ABA + AB^2$

$= A^3 - B^3 - A^2B + A^2B = A^3 - B^3$

43. (c) : $AB^2 = BA$

$\therefore \quad A^{-1}AB^2 = A^{-1}BA$

$\Rightarrow \quad A^{-1}BA = B^2$ and $A^6 = I$

$A^{-1}BA = B^2 \Rightarrow (A^{-1}BA)^6 = B^{12}$

$\Rightarrow \quad B^6 = B^{12} \Rightarrow B^6 = 1 \Rightarrow B^{127} = I$

44. (a) : $(A^3 + B^3 + C^3)(A - B)$

$= A^4 - A^3B + B^3A - B^4 + C^3A - C^3B = 0$

$|(A^3 + B^3 + C^3)(A - B)| = 0$

$\Rightarrow \quad |(A^3 + B^3 + C^3)| = 0$, since $|(A - B)| \neq 0$.

45. (a) : $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\Rightarrow \quad U_1 = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$U_2 = A^{-1}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$$

$$U_3 = A^{-1}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$$

$$\therefore \quad U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \Rightarrow \det U = 3$$

46. (b) : $U^{-1} = \dfrac{1}{3}\begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$

Sum of the elements is 0.

47. (a) : $\begin{bmatrix} 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} 7 \\ -8 \\ -5 \end{bmatrix} = 5$$

48. (c) :

$$AA^* = KK\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} = K^2\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = I$$

$\Rightarrow \quad 3K^2 = 1 \Rightarrow K = \pm\dfrac{1}{\sqrt{3}}$

49. (b) : $A^*Q^{20}A = A^*(ABA^*)Q^{19}A$

$= IBA^* \ ABA^*Q^{18}A$

$= B^2A^*Q^{18}A = \ldots\ldots\ldots = B^{20}$

50. (a) : $\det(\text{adj}(\text{adj}X)) = (\det X)^{(n-1)^2} = 1^1 = 1$

$(\because \det|X| = 1)$

51. A – q, r, s; B – p; C – q, r; D – p

Here $D = 0$ and $D_x = D_y = D_z = 0 \Rightarrow 3a + b - c = 0$

52. A – r, s; B – p, q; C – p, q; D – p, q;

$8 + a + b = k, 13 + e + f = k, 10 + c = k, 11 + d = k$

$\Rightarrow \quad c = 9, d = 8, a + b = 6 + 5$ or $5 + 6, e + f = 2 + 4$ or $4 + 2$.

53. A – r, s; B – p, q; C – p, q; D – p, q;

(A)

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \Rightarrow A^t = A \ \& \ AA^T = 9I \Rightarrow A^2 = 9I$$

Again, cofactor matrix of $A = \begin{pmatrix} -3 & -6 & -6 \\ -6 & -3 & 6 \\ -6 & 6 & -3 \end{pmatrix} = -3A$

$\therefore$ adjoint of A = transpose of cofactor matrix of A
= cofactor matrix of A = $-3A$

(B) $3B = A \Rightarrow B = (1/3)A$ But $AA^T = A^TA = 9I$

$\therefore$ A is orthogonal & involutory matrix so the matrix B.

(C) $B = \frac{1}{3}A \Rightarrow B^T = \frac{1}{3}A^T$

Also $BB^T = B^TB = \frac{1}{9}A^2 = I$

(D) Again $B = (1/3)A$ where $(1/3)A$ is involutory & orthogonal.

54. A – q; B – s; C – r; D – p

$P(\theta) = \sqrt{2}\cos 2\theta + \sin 2\theta \Rightarrow$ Range $= [0, 2\sqrt{2}]$

$\because \sqrt{a^2+b^2-c} \le a\sin x + b\sin x + c \le \sqrt{a^2+b^2+c}$

$$q(\theta) = \begin{vmatrix} \sin 2\theta & 0 & 1 \\ \cos 2\theta & 1 & -3 \\ 2 & 2 & -5 \end{vmatrix} = \sin 2\theta + 2\cos\theta - 2,$$

Range $[-2-\sqrt{5}, -2+\sqrt{5}]$

Applying $C_1 \to C_1 + C_3$, we get

$$r(\theta)\begin{vmatrix} 2\cos\theta & \sin\theta & \cos\theta \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} = 2\cos\theta, \text{ Range} = [-2, 2]$$

$s(\theta) = \sec^2\theta.\cos^2\theta(\cot^2\theta - \text{cosec}^2\theta) - \cos^2\theta\ (\cot^2\theta - \text{cosec}^2\theta] + \cos^2\theta(\cos^2\theta - 1)$

$= -1 + 1 + \cos^2\theta - \cos^2\theta\sin^2\theta = \cos^4\theta$

$\Rightarrow$ Range $= [0, 1]$

55. A – p; B – r; C – s; D – p;

(A) Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = x\begin{vmatrix} 1 & 1 & 1 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} \Rightarrow x = 0$$

(B) $$A = \begin{vmatrix} a & a & a \\ 3a & 4a & 5a \\ 6a & 9a & 11a \end{vmatrix} = a^3\begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 6 & 9 & 11 \end{vmatrix} = i$$

(C) Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\Rightarrow \quad \Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} = 3\sqrt{3}\,i$$

(D) Sub: $x = 1$ on both sides $A + B + C + D + E = 0$

56. (1) : $$\begin{vmatrix} 1 & -2 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow -3 + 0 + 3\lambda = 0 \Rightarrow \lambda = 1$$

57. (7) : We have

$$\sum_{k=1}^{n}\Delta_k = \begin{vmatrix} \sum_{k=1}^{n}1 & n & n \\ \sum_{k=1}^{n}2k & n^2+n+1 & n^2+n \\ \sum_{k=1}^{n}(2k-1) & n^2 & n^2+n+1 \end{vmatrix}$$

$$= \begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n^2 & n^2+n+1 \end{vmatrix}$$

$$= \begin{vmatrix} n & 0 & 0 \\ n(n+1) & 1 & 0 \\ n^2 & 0 & n+1 \end{vmatrix} = n(n+1) = 56 = 7\times 8$$

58. (1) : Expanding $S^3 + 2C^3 - 3SC^2 = 0$
where $S = \sin x$, $C = \cos x$

$(S - C)(S + C)(S - 2C) = 0$

since $\frac{-\pi}{4} \le x \le \frac{\pi}{4}$, the only solution is $S - C = 0$ or $\tan x = 1$

59. (3) : Let Δ be the given determinant. Then apply column transformation $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$, we get

$$\Delta = \begin{vmatrix} x & 0 & 1 \\ y & 2y & 1 \\ -2z & -2z & 1+3z \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} x & 0 & 0 \\ y & 2y & 1-\frac{y}{x} \\ -2z & -2z & 1+3z+\frac{2z}{x} \end{vmatrix} = 0 \text{ (by } C_3 \to C_3 - \frac{1}{x}C_1)$$

Solving we get

$$x\left[2y\left(1+3z+\frac{2z}{x}\right)+2z\left(1-\frac{y}{x}\right)\right]=0$$

$$\left[2xy+6zxy+4yz+2zx-2yz\right]=0$$

$$2(xyz)\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+3\right]=0 \qquad -\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=3$$

60. (3) : For nilpotent matrix $A^k = 0$ and $A^{k-1} \neq 0$

Since $\begin{bmatrix} xy & x^2 \\ -y^2 & -xy \end{bmatrix}$ is always nilpotent,

$\alpha^2 = 9 \Rightarrow \alpha = 3, -3$ but $\alpha > 0 \therefore \alpha = 3$

61. (2) : Applying $C_1 \to C_1 + C_2$, we get

$$\begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2+4\sin 4\theta = 0 \Rightarrow \sin 4\theta = \frac{-1}{2}$$

$$\therefore \quad \sin 4\theta = \sin 210°, \sin 330°$$

$$\therefore \quad \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$$

Number of solutions are 2 in $\left(0, \frac{\pi}{2}\right)$

62. (2) : $R_3 \to R_3 - R_2 - 8R_1$, R_3 becomes (0, 0, 0)

63. (3) : Clearly $f'(x) = 0 \Rightarrow f(x) = K \ \forall \ x$

$$\therefore \quad f(2) = \frac{1}{4} \Rightarrow f(x) = \frac{1}{4} \ \forall \ x$$

$$\sum_{r=1}^{15} f(x_r) = \frac{15}{4} \Rightarrow \left[\frac{15}{4}\right] = 3$$

64. (5) : $\begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow \lambda = 5$

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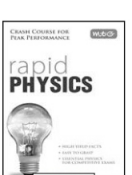

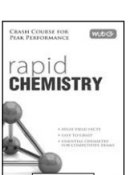

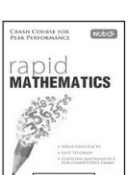

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XI

ORDERED PAIR

A pair of objects (elements) taken (listed) in a definite (specific) order is known as an ordered pair.

CARTESIAN PRODUCT OF SETS

Let X and Y be two non-empty sets, then the set of all distinct ordered pairs of the form (x_i, y_i) is called cartesian product of X and Y. We denote the cartesian product of X and Y by $X \times Y$, *i.e.*,
$X \times Y = (x_i, y_i)$, where $x_i \in X, y_i \in Y$

Remark :

(i) In ordered pairs (x_i, y_i), x_i is known as first element and y_i is known as second element.
Two ordered pairs (x, y) and (p, q) are said to be equal $\Leftrightarrow x = p, y = q$.

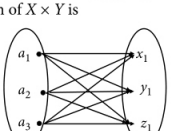
(ii) If there are p elements in X and q elements in Y, then there will be pq elements in $X \times Y$ *i.e.*, if $n(X) = p$ and $n(Y) = q$, then $n(X \times Y) = pq$

(iii) If X and Y are non-empty sets and either X or Y is an infinite set, then so is $X \times Y$.

(iv) $X \times X \times X = \{(a, b, c) : a, b, c \in X\}$. Here (a, b, c) is called an ordered triplet.

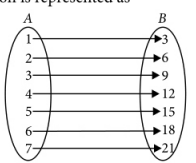
ARROW DIAGRAM OF CARTESIAN PRODUCT OF TWO SETS

If X and Y be two sets then to represent $X \times Y$ by an arrow diagram we draw venn diagrams representing set X and Y one opposite to the other.
Let $X = \{a_1, a_2, a_3\}$ & $Y = \{x_1, y_1, z_1\}$,
then diagram of $X \times Y$ is



RELATIONS

It is an association between, or property of two or more objects.
For example, let A and B be two sets, such that $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 9, 12, 15, 18, 21\}$ then there exist a relation "third multiple of" between the elements of A and B, then diagrammatically the given relation is represented as



and also can be written in ordered pairs as
$R = \{(1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21)\}$
Above relation indicates that element of B are third multiples of elements of A. Also, we observe that above relation is a subset of $A \times B$.

Definition of Relation : Let X and Y be two non-empty sets. Then subset of $X \times Y$ is called a **relation** from X to Y. The subset is obtained by describing a relationship between the first element and the second element of the ordered pairs in $X \times Y$. The second element is called the image of the first element.

- **Domain in Relation :** The set of all first elements of the ordered pairs in relation R from set X to set Y is called the domain of the relation R.
- **Range in Relation :** The set of all second elements of the ordered pairs in relation R from set X to set Y is called the range of the relation R.

- **Co-domain in Relation :** The whole set Y is called the co-domain of the relation R.

  Range $\subseteq$ Co-domain

FUNCTIONS

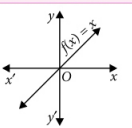
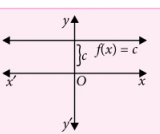
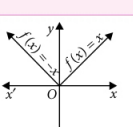
A relation f from a set X to a set Y is said to be a **function** if every element of set X has one and only one image in set Y.

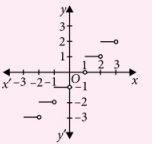
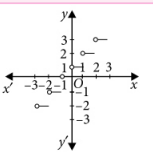
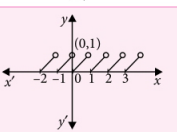
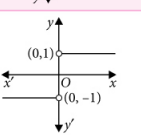
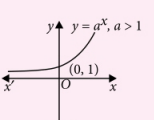
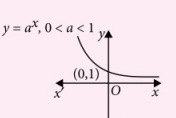
In other words, a function f is a relation such that no two pairs in the relation has the same first element. The notation $f : X \to Y$ means that f is a function from X to Y. X is called the domain of f and Y is called the co-domain of f. Given an element $x \in X$, there is a unique element y in Y that is related to x. The unique element y to which f relates x is denoted by $f(x)$ and is called f of x, or the image of x under f. The set of all values of $f(x)$ taken together is called the range of f or image of X under f, *i.e.*,

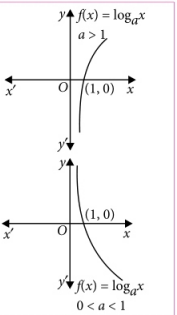
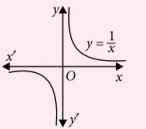
Range of $f = \{y \in Y | y = f(x)$, for some x in $X\}$.

Definition : A function which has either R or one of its subsets as its range, is called a **real valued function.** Further, if its domain is also either R or a subset of R, it is called a **real function.**

TYPES OF FUNCTIONS

S.No.	NAME	DEFINITION	DIAGRAM
1.	Identity Function	The function $f : R \to R$ defined by $y = f(x) = x$ for each $x \in R$. Domain of $f = R$. Range of $f = R$.	
2.	Constant Function	The function $f : R \to R$ defined by $y = f(x) = c$, $x \in R$, where c is a constant $\in R$. Domain of $f = R$. Range of $f = \{c\}$.	
3.	Polynomial Function	A real valued function $f : R \to R$ defined by $y = f(x) = a_0 + a_1x + \ldots\ldots + a_nx^n$, where $n \in N$ and $a_0, a_1, a_2, \ldots, a_n \in R$ for each $x \in R$.	
4.	Rational Function	These are the real functions of the type $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of x defined in a domain, where $g(x) \neq 0$.	
5.	Modulus Function	The real function $f : R \to R$ defined by $f(x) = \|x\| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases} \forall x \in R$. Domain of $f = R$. Range of $f = R^+ \cup \{0\}$.	

6.	Greatest Integer Function (Floor Function)	The real function $f : R \to R$ defined by $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x. Domain of $f = R$. Range of $f = Z$.	
7.	Smallest Integer Function (Ceiling or Step Function)	The real function $f : R \to R$ defined by $f(x) = \lceil x \rceil \; \forall \, x \in R$, assumes the value of the least integer, greater than or equal to x. Domain of $f = R$. Range of $f = Z$.	
8.	Fractional Part Function	The function $f : R \to R$ defined by $f(x) = \{x\} \; \forall \, x \in R$. Domain of $f = R$. Range of $f = [0, 1)$.	
9.	Signum Function	The real function $f : R \to R$ defined by $f(x) = \begin{cases} \dfrac{\lvert x \rvert}{x}, x \neq 0 \\ 0, \; x = 0 \end{cases} = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{ if } x < 0 \end{cases}$ Domain of $f = R$. Range of $f = \{-1, 0, 1\}$.	
10.	Exponential Function	A real function $f : R \to R$ defined by $f(x) = a^x$, where $a > 0$ and $a \neq 1$ Domain of $f = R$ Range of $f = (0, \infty)$ **Case I :** When $a > 1$ We observe that the values of $y = f(x) = a^x$ increase as the values of x increase. Also, $f(x) = a^x \begin{cases} < 1, \text{ for } x < 0 \\ = 1, \text{ for } x = 0 \\ > 1, \text{ for } x > 0. \end{cases}$ **Case II :** When $0 < a < 1$ In this case, the values of $y = f(x) = a^x$ decrease with the increase in x and $y > 0 \; \forall \; x \in R$. Also, $y = f(x) = a^x \begin{cases} > 1, \text{ for } x < 0 \\ = 1, \text{ for } x = 0 \\ < 1, \text{ for } x > 0 \end{cases}$	 

11.	Logarithmic Function	If $a > 0$ and $a \neq 1$, then the function defined by $f(x) = \log_a x, x > 0$. Domain of $f = (0, \infty)$. Range of $f = R$. **Case I :** When $a > 1$ $y = f(x) = \log_a x = \begin{cases} < 0, \text{for } 0 < x < 1 \\ = 0, \text{for } x = 1 \\ > 0, \text{for } x > 1 \end{cases}$ **Case II :** When $0 < a < 1$ $y = f(x) = \log_a x = \begin{cases} > 0, \text{for } 0 < x < 1 \\ = 0, \text{for } x = 1 \\ < 0, \text{for } x > 1 \end{cases}$	
12.	Reciprocal Function	The real function $f : R - \{0\} \to R$ defined by $f(x) = \frac{1}{x}$ Domain of $f = R - \{0\}$. Range of $f = R - \{0\}$.	

ALGEBRA OF REAL FUNCTIONS

(i) Addition of two real functions

Let $f : X \to R$ and $g : X \to R$ be any two real functions, where $X \subset R$, then $(f + g) : X \to R$ defined as $(f + g)(x) = f(x) + g(x)$, for all $x \in X$.

(ii) Subtraction of real function from another

Let $f : X \to R$ and $g : X \to R$ be two functions, where $X \subset R$, then $(f - g) : X \to R$ defined as $(f - g)(x) = f(x) - g(x)$, for all $x \in X$.

(iii) Multiplication by a Scalar

Let $f : X \to R$ be real valued function and α be a scalar, then $\alpha \cdot f : X \to R$ defined as $(\alpha f)(x) = \alpha f(x)$, for all $x \in X$.

(iv) Multiplication of two real functions

Let $f : X \to R$ and $g : X \to R$ be two real functions then $fg : X \to R$ defined as $(fg)(x) = f(x)\, g(x)$, for all $x \in X$.

(v) Quotient of two real functions

Let $f : X \to R$ and $g : X \to R$ be two real functions, then $\frac{f}{g} : X \to R$ defined as $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0, x \in X$

VERY SHORT ANSWER TYPE (1 MARK)

1. If $(3x, x + y) = (6, 3)$, find x, y.
2. Let $X = \{-1, 0\}$ and $Y = \{2, 5\}$, then prove that $X \times Y \neq Y \times X$.
3. Let $X = \{a, b, c\}$ and $Y = \phi$. Find the number of ordered pairs in $X \times Y$.
4. Let $A = \{a_1, a_2\}$ and $B = \{b_1, b_2\}$ then find the number of relations from A to B.
5. If $f : R \to R$ is defined by $f(x) = \frac{x}{x^2 + 1}$, find $f(f(2))$.
6. If R is the relation "is less than" from $A = \{1, 2, 3, 4, 5\}$ to $B = \{1, 4, 5\}$, then write down the solution of R. Also find the inverse relation of R.
7. Which of the following relations are functions from $A = \{1, 3, 5, 7, 9\}$ to $B = \{1, 2, 3, 4, 5\}$?
   (i) $f = \{(3, 1), (5, 1), (7, 1), (9, 1)\}$
   (ii) $f = \{(3, 2), (1, 5), (5, 1), (7, 4), (9, 5)\}$

SHORT ANSWER TYPE (4 MARKS)

8. Find the domain and range of the function $f(x)$ given by $f(x)=\dfrac{x-2}{3-x}$.

9. Find the domain, range of $f(x)=\dfrac{3}{2-x^2}$

10. Find the domain and range of the function:

$$f=\left\{x:\left(\frac{1}{1-x^2}\right):x\in R, x\neq \pm 1\right\}$$

11. Let $f(x) = x^3$ and $g(x) = x^2 + 1$ be two real functions. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, $\left(\dfrac{f}{g}\right)(x)$.

12. Let $f(x)=\sqrt{x}-x$ and $g(x)=x^2$ be two functions defined over the set of non-negative real numbers. Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$ and $\left(\dfrac{f}{g}\right)(x)$.

13. Find the sum and difference of the identity function and the modulus function.

14. Let $f : [2, \infty) \to R$ and $g : [-2, \infty) \to R$ be two real functions defined by $f(x)=\sqrt{x-2}$ and $g(x)=\sqrt{x+2}$. Find $f + g$ and $f - g$.

15. (i) If the graph of a function 'f' does not meet the first and the third quadrants, what is the corresponding property of that function.

(ii) If for a function 'f', $f(1 + x) = f(1 - x)$ for all $x \in R$, then how is this property reflected in its graph?

(iii) If f and g are two functions such that $D_f = D_g$ and $f(x) \le g(x)$, $\forall\ x \in D_f$ (or D_g), then how is this property reflected in its graph?

SOLUTIONS

1. Given $(3x, x + y) = (6, 3)$

$\Rightarrow \quad 3x = 6 \quad ...(1)$

and $\quad x + y = 3 \quad ....(2)$

From (1), $x = 2$ and then from (2), $y = 1$.

2. $X \times Y = \{-1, 0\} \times \{2, 5\}$

$= \{(-1, 2), (-1, 5), (0, 2), (0, 5)\}$

$Y \times X = \{2, 5\} \times \{-1, 0\}$

$= \{(2, -1), (2, 0), (5, -1), (5, 0)\}$

Hence $X \times Y \neq Y \times X$.

3. $X \times Y = \{a, b, c\} \times \{\ \} = \phi$

Hence there are no ordered pairs formed in $X \times Y$.

4. $n(A) = 2, n(B) = 2 \Rightarrow n(A \times B) = 4$

$\therefore$ Number of relations $= 2^{pq} = 2^4 = 16$

5. Given $f(x)=\dfrac{x}{x^2+1}\Rightarrow f(2)=\dfrac{2}{2^2+1}=\dfrac{2}{5}$

$$\Rightarrow f(f(2))=f\left(\frac{2}{5}\right)=\frac{\frac{2}{5}}{\left(\frac{2}{5}\right)^2+1}=\frac{\frac{2}{5}}{\frac{4}{25}+1}=\frac{\frac{2}{5}}{\frac{29}{25}}=\frac{10}{29}.$$

6. Given $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 4, 5\}$ and R is the relation "is less than" from A to B, then $R = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$.

Further, $R^{-1} = \{(y, x) : y \in B, x \in A, (x, y) \in R\}$, therefore,

$R^{-1} = \{(4, 1), (5, 1), (4, 2), (5, 2), (4, 3), (5, 3), (5, 4)\}$ and it is the relation "is greater than" from B to A.

7. **(i)** is not a function as $f(1)$ is not defined, *i.e*; there is no ordered pair in f whose first element is 1.

(ii) is a function.

8. We have, $f(x)=\dfrac{x-2}{3-x}$

Domain of f : Clearly, $f(x)$ is defined for all x satisfying $3 - x \neq 0$ *i.e.* $x \neq 3$.

Hence, Domain $(f) = R - \{3\}$.

Range of f: Let $y = f(x)$, *i.e.* $y=\dfrac{x-2}{3-x}$

$\Rightarrow\ 3y - xy = x - 2 \Rightarrow x(y + 1) = 3y + 2$

$\Rightarrow\ x=\dfrac{3y+2}{y+1}$

Clearly, x assumes real values for all y except $y + 1 = 0$ *i.e.* $y = -1$.

Hence, Range $(f) = R - \{-1\}$.

9. We have, $f(x)=\dfrac{3}{2-x^2}$

Domain of f:

For $f(x)$ to be real, we must have

$2 - x^2 \neq 0 \Rightarrow x \neq \pm\sqrt{2}$

$\therefore$ Domain $(f) = R - \{-\sqrt{2}, \sqrt{2}\}$

Range of f:

Let $f(x) = y$. Then, $y = \dfrac{3}{2-x^2} \Rightarrow 2y - x^2y = 3$

$\Rightarrow x^2y = 2y - 3 \Rightarrow x = \pm\dfrac{\sqrt{2y-3}}{y}$

Now, x will take real values other than $-\sqrt{2}$ and $\sqrt{2}$, if

$\dfrac{2y-3}{y} \geq 0$

$\Rightarrow y \in (-\infty, 0) \cup \left[\dfrac{3}{2}, \infty\right)$

Hence, range $(f) = (-\infty, 0) \cup [3/2, \infty)$.

10. We have, $f(x) = \dfrac{1}{1-x^2}$

Domain of f: Clearly, $f(x)$ is defined for all $x \in R$ except for which $x^2 - 1 = 0$ i.e., $x = \pm 1$.

Hence, Domain of $f = R - \{-1, 1\}$.

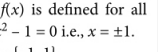
Range of f: Let $f(x) = y$. Then,

$\dfrac{1}{1-x^2} = y \Rightarrow 1 - x^2 = \dfrac{1}{y}$

$\Rightarrow x^2 = 1 - \dfrac{1}{y} = \dfrac{y-1}{y} \Rightarrow x = \pm\sqrt{\dfrac{y-1}{y-0}}$

Clearly, x will take real values, if

$\dfrac{y-1}{y-0} \geq 0$



$\Rightarrow y < 0$ or $y \geq 1$

$\Rightarrow y \in (-\infty, 0) \cup [1, \infty)$

Hence, range $(f) = (-\infty, 0) \cup [1, \infty)$

11. $(f+g)(x) = f(x) + g(x) = x^3 + x^2 + 1$

$(f-g)(x) = f(x) - g(x) = x^3 - x^2 - 1$

$(fg)(x) = f(x)\, g(x) = x^3(x^2 + 1) = x^5 + x^3$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3}{x^2+1}$

12. $(f+g)(x) = f(x) + g(x) = \sqrt{x} - x + x^2$

$(f-g)(x) = f(x) - g(x) = \sqrt{x} - x - x^2$

$(fg)(x) = f(x)g(x) = (\sqrt{x} - x)x^2 = x^{5/2} - x^3$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x} - x}{x^2} = \dfrac{\sqrt{x}}{x^2} - \dfrac{1}{x}$

13. We know that $f : R \to R$ defined by $f(x) = x$ is the identity function and $g : R \to R$ defined by $g(x) = |x|$ is the modulus function.

Clearly, f and g have the same domain.

Now, $(f+g)(x) = f(x) + g(x)$

$\Rightarrow (f+g)(x) = x + |x|$

$\Rightarrow (f+g)(x) = \begin{cases} x + x, & \text{if } x \geq 0 \\ x - x, & \text{if } x < 0 \end{cases}$

$\Rightarrow (f+g)(x) = \begin{cases} 2x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$

and,

$(f-g)(x) = f(x) - g(x)$

$\Rightarrow (f-g)(x) = x - |x|$

$\Rightarrow (f-g)(x) = \begin{cases} x - x, & \text{if } x \geq 0 \\ x - (-x), & \text{if } x < 0 \end{cases}$

$\Rightarrow (f-g)(x) = \begin{cases} 0, & \text{if } x \geq 0 \\ 2x, & \text{if } x < 0 \end{cases}$

14. Here, we have

$D_1 = [2, \infty)$ and $D_2 = [-2, \infty)$

$\therefore \quad D_1 \cap D_2 = [2, \infty)$

Thus, $f + g : [2, \infty) \to R$ and $f - g : [2, \infty) \to R$ are given by

$(f+g)(x) = f(x) + g(x) = \sqrt{x-2} + \sqrt{x+2}$ for all $x \in [2, \infty)$

and $(f-g)(x) = f(x) - g(x) = \sqrt{x-2} - \sqrt{x+2}$ for all $x \in [2, \infty)$

15. (i) $xf(x) \leq 0$ for all $x \in D_f$

(ii) Graph is symmetrical about the line $x = 1$.

(iii) Graph of 'f' at any point either coincides with that of g or lies below it.

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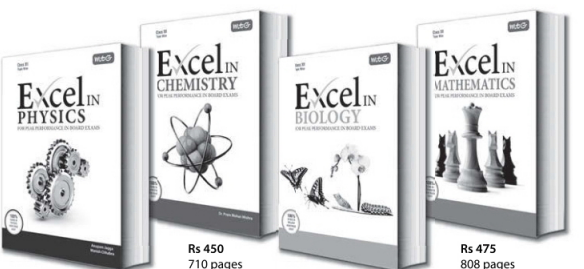

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MATHS MUSING

SOLUTION SET-138

1. (a) : $M=\begin{pmatrix}12\\6\end{pmatrix}, N=\begin{pmatrix}12-1\\6-1\end{pmatrix}=\begin{pmatrix}11\\5\end{pmatrix}$

$M-N=\begin{pmatrix}12\\6\end{pmatrix}-\begin{pmatrix}11\\5\end{pmatrix}=\begin{pmatrix}11\\6\end{pmatrix}=\begin{pmatrix}11\\5\end{pmatrix}=462$

2. (c): $I=\dfrac{e^{-2x}}{-2}(x^4-2x^3+4x^2-2x-1)\Big|_0^\infty$

$=-\dfrac{e^{-2x}}{4}(4x^3-6x^2+8x-2)\Big|_0^\infty+\dfrac{e^{-2x}}{-8}(12x^2-12x+8)\Big|_0^\infty$

$=-\dfrac{e^{-2x}}{16}(24x-12)\Big|_0^\infty+\dfrac{e^{-2x}}{-32}\cdot 24\Big|_0^\infty$

$=-\dfrac{1}{2}-\dfrac{1}{2}+1-\dfrac{3}{4}+\dfrac{3}{4}=0$

3. (c): $D=\dfrac{\pi}{2}-\dfrac{A}{2}, E=\dfrac{\pi-B}{2}, F=\dfrac{\pi-C}{2}$

$\dfrac{\Delta DEF}{\Delta ABC}=\dfrac{2r^2\sin D\sin E\sin F}{2R^2\sin A\sin B\sin C}=\dfrac{r}{2R}$

4. (d) : $|x-y|\geq 6$ for the 10 pairs : (1, 7), (1, 8), (1, 9), (1, 10), (2, 8), (2, 9), (2, 10), (3, 9), (3, 10), (4, 10)

Required probability $=1-\dfrac{10}{\begin{pmatrix}10\\2\end{pmatrix}}=1-\dfrac{2}{9}=\dfrac{7}{9}$

5. (a) : $x+iy=re^{i\theta}, \dfrac{x+iy}{x-iy}=e^{2i\theta}$

$\cot[-i\ln e^{2i\theta}]=\cot 2\theta=\dfrac{1}{\tan 2\theta}=\dfrac{x^2-y^2}{2xy}$

6. (a, c): The plane is $2x-y+z-3+\lambda(3x+y+z-5)=0$

$(2+3\lambda)x+(\lambda-1)y+(\lambda+1)z-(3+5\lambda)=0$

The distance of (2, 1, –1) from it is $\dfrac{1}{\sqrt{6}}$

$\Rightarrow \lambda=0, \lambda=-24.$

The planes are $2x-y+z=3, 62x+29y+19z=105$

$\therefore \quad a+b+c=110, 70$

7. (d) : The curves meet at $\left(\dfrac{2}{\sqrt{3}},\sqrt{\dfrac{2}{3}}\right)$ in the first quadrant. The common area is

$$4\int_0^{2/\sqrt{3}}\frac{1}{2}\sqrt{4-x^2}\,dx+\int_{2/\sqrt{3}}^{\sqrt{2}}\sqrt{2-x^2}\,dx=8\tan^{-1}\frac{1}{\sqrt{2}}$$

$=4\tan^{-1}2\sqrt{2}$

8. (a) : The tangent to the ellipse at $P(2\cos\theta, \sin\theta)$ is also tangent to the circle for $P\left(2\sqrt{\dfrac{2}{3}},\dfrac{1}{\sqrt{3}}\right)$. The length of common tangent $=\sqrt{OP^2-2}=\sqrt{3-2}=1$

9. (7) : $iz=-\ln(7-4\sqrt{3})=\ln(7+4\sqrt{3})$

$\therefore \quad \cos z=\dfrac{e^{iz}+e^{-iz}}{2}=\dfrac{7+4\sqrt{3}+7-4\sqrt{3}}{2}=7$

10. (d) : $C=\dfrac{\pi}{2}, r_3=s\tan\dfrac{C}{2}=s$

$\Delta^2=r_1r_2r_3r=r_1r_2sr\Rightarrow\Delta=M_1r_2=2$

$s-a=1, s-b=2, c=3, a-b=1$

$ab=2\Delta=4\Rightarrow 2a=\sqrt{17}+1, 2b=\sqrt{17}-1$

$2r_3=2s=a+b+c=\sqrt{17}+3$

$2r=\dfrac{2\Delta}{s}=\dfrac{4\Delta}{2s}=\dfrac{8}{\sqrt{17}+3}=\sqrt{17}-3$

Solution Sender of Maths Musing

SET-137

1.	Naresh Ch. Biswas	: Kolkata, W.B.

SET-138

1.	Devjit Acharjee	: Kolkata, W.B.
2.	Chandan Pramanik	: Medinipur, W.B.
3.	Khwaja Sami Baig	: Azamgarh, U.P.

MOCK TEST PAPER 2Q15

JEE Main & Advanced

Series-1

Sequences & Series : Arithmetic, Geometric and Harmonic progressions, arithmetic, geometric, and harmonic means, sums of finite arithmetic and geometric progression, infinite geometric series, sums of squares and cubes of first n natural numbers.

Trigonometry : Trigonometric functions, their periodicity and graphs, addition and subtraction formulae, formulae involving multiple and sub-multiple angles.

Coordinate Geometry-2D : Cartesian coordinates, distance between two points, section formulae, shift of origin. Locus problems.

PAPER-I

Time : 2 hrs. Marks : 90

SECTION-I

Single Option Correct

[3 marks for correct answer and 1 for wrong answer]

This section contains 10 multiple choice questions. Each question has 4 choices, out of which ONLY ONE is correct.

1. If the sum of the n terms of an A.P. is $cn(n-1)$ where $c \neq 0$, then the sum of the squares of these terms is

(a) $c^2n^2(n+1)^2$ (b) $\frac{2}{3}c^2n(n-1)(2n-1)$

(c) $\frac{2}{3}c^2n(n+1)(2n+1)$ (d) $\frac{1}{3}c^2n^2$

2. If a, b, c are three distinct real numbers, which are in G.P. and $a+b+c=xb$. Then

(a) $x<-1$ or $x>3$ (b) $-1<x<3$

(c) $-1<x<2$ (d) $0<x<1$

3. If three unequal numbers a, b, c are in H.P. and their squares are in A.P., the $a:b:c=$

(a) $1+\sqrt{3}:-2:1-\sqrt{3}$ (b) $1:2:3$

(c) $\sqrt{3}+1:\sqrt{3}-1:2$ (d) $3:2:1$

4. If $f(x) = \cos[\pi]x + \cos[\pi x]$, where $[x]$ is the greatest integer function of x, then $f\left(\frac{\pi}{2}\right)=$

(a) 0 (b) cos 3

(c) cos 4 (d) none of these

5. If in a triangle ABC, $C > \frac{3\pi}{4}$, then the value of $(1+\tan A)(1+\tan B)$ will lie in the interval

(a) (1, 2) (b) (0, 1)

(c) (2, 8) (d) none of these

6. If $a\sin x + b\cos(x+\theta) + b\cos(x-\theta) = d$ for some real x, the minimum value of $|\cos\theta|$ is equal to

(a) $\frac{1}{2|b|}\sqrt{d^2-a^2}$ (b) $\frac{1}{2|a|}\sqrt{d^2-a^2}$

(c) $\frac{1}{2|d|}\sqrt{d^2-a^2}$ (d) none of these

7. If $\theta=\frac{\pi}{7}$, then the value of

$(\tan\theta\cdot\tan2\theta+\tan2\theta\cdot\tan4\theta+\tan4\theta\cdot\tan\theta)$ is

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $-\frac{1}{7}$ (d) -7

8. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on a

(a) straight line (b) circle

(c) ellipse (d) none of these

9. The coordinate of the point on the y-axis which is equidistant from the points $(-5, 4)$ and $(3, -2)$ is

(a) $\left(0, \frac{4}{3}\right)$ (b) $\left(0, \frac{3}{4}\right)$

(c) $\left(0, \frac{7}{3}\right)$ (d) $(0, 1)$

By : Sankar Ghosh, HOD(Math), Takshyashila. Mob : 09831244397.

10. The area of the triangle with vertices $(p+1, 1)$ $(2p+1, 3)$ and $(2p+2, 2p)$ is 9 square units. Then the integral value of p is equal to

(a) 4 (b) 1

(c) 2 (d) 3

SECTION-II

Multiple Correct Options

[4 marks for correct answer and 1 for wrong answer]

This section contains 5 multiple choice questions. Each question has 4 choices, out of which ONE or MORE is/are correct.

11. If $5x-y$, $2x+y$, $x+2y$ are in A.P. and $(x-y)^2$, $(xy+1)$, $(y+1)^2$ are in G.P., $x \neq 0$, then $x+y=$

(a) $\frac{3}{4}$ (b) 3 (c) -5 (d) -6

12. If $3(\cos 2\varphi - \cos 2\theta) = 1 - \cos 2\theta \cos 2\varphi$ and $\tan\theta = k\tan\varphi$, then the value of k is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $-\frac{1}{\sqrt{2}}$ (d) $-\sqrt{2}$

13. If $n \in I$ and $f_n(\theta) = \tan\left(\frac{\theta}{2}\right)(1+\sec\theta)$ $(1+\sec 2\theta)(1+\sec 4\theta)...(1+\sec 2^n\theta)$ then

(a) $f_2\left(\frac{\pi}{16}\right)=1$ (b) $f_3\left(\frac{\pi}{32}\right)=1$

(c) $f_4\left(\frac{\pi}{64}\right)=1$ (d) $f_5\left(\frac{\pi}{128}\right)=1$

14. The point P divides the line segment joining the points $A(-5, 1)$ and $B(3, 5)$ in $\lambda : 1$. The coordinates of the points Q and R are respectively $(1, 5)$ and $(7, 2)$. If the area of the triangle ΔPQR is 2 square units then the value of λ is

(a) $\frac{19}{5}$ (b) $\frac{31}{9}$ (c) 23 (d) 19

15. If the vertices of the triangle PQR are rational points, which of the following points of the triangle PQR are always rational points?

(a) Centroid (b) Incentre

(c) Circumcentre (d) Orthocentre

SECTION-III

Assertion & Reason Type

[4 marks for correct answer and 1 for wrong answer]

This section contains 4 questions. Each question contains Statement-1 (assertion) and Statement-2 (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

In each of the following questions two statements are given as Assertion (A) and Reason (R). Examine the statements carefully and answer the questions according to the instruction given below.

(a) if both A and R is correct and R is the proper reason of A.

(b) if both A and R is correct and R is not the proper reason of A.

(c) If A is correct and R is wrong.

(d) If A is wrong and R is correct.

16. **Statement 1 :** The A.M. of two numbers is $\frac{5}{2}$ the G.M. is 2 and H.M. is $\frac{8}{5}$.

Statement 2 : For any numbers G.M.^{2} = A.M. $\times$ H.M.

17. **Statement 1 :** $1^3 - 2^3 + 3^3 - 4^3 + ... + 11^3 = 378$

Statement 2 : For any odd integer, where $n \geq 1$, $n^3 - (n-1)^3 + ... + (-1)^{n-1} \cdot 1^3 = \frac{1}{4}(2n-1)\cdot(n+1)^2$

18. **Statement 1 :** If $x+y+z=xyz$, then any one of them is negative.

Statement 2 : For any triangle, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ then any one of the angles is obtuse.

19. **Statement 1 :** If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$ then $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$

Statement 2 : If $a^2 + b^2 = 0$, then $a = b = 0$

20. **Statement 1 :** Let $A(a\cos t, a\sin t)$, $B(b\sin t, -b\cos t)$, $C(1, 0)$ are the vertices of the triangle ΔABC and t is a variable parameter, then the locus of the centroid is $(3x-1)^2 + (3y)^2 = a^2 + b^2$

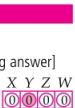
Statement 2 : The centroid of any triangle is equidistant from its vertices.

SECTION-IV

Integer Answer type

[3 marks for correct answer and 1 for wrong answer]

This section contains 10 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following.



21. If the 5th term of a G.P. is 2 and the product of 9 terms of the progression is $128k$, then the value of k is

22. The sum of three terms of a G.P. is 14. If 1 is added to 1st & 2nd term and 1 is subtracted from 3rd term the newly formed three numbers are in A.P. The smallest number is

23. If $A + B = 225°$ then $\frac{1+\cot A}{\cot A}\cdot\frac{1+\cot B}{\cot B} =$

24. If $x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right)$ then the value of $(xy + yz + zx)$ is

25. If $A(0, \sqrt{2})$ and $B(2\sqrt{2}, 0)$ are two vertices of ΔABC and $AC = BC$ and if the equation of BC is $x = 2\sqrt{2}$, then the area of the triangle is

PAPER-II

Time : 2 hrs. Marks : 90

SECTION-I

Single Option Correct

This section contains 10 multiple choice questions numbered 1 to 4. Each question has 4 choices (a), (b), (c)and (d), out of which **ONLY ONE** is correct. [3 marks for correct answer and –1 for wrong answer]

1. If a, b, c are positive numbers in A.P., such that their product is 64, then the minimum value of b is

(a) 2 (b) 4
(c) 1 (d) does not exist

2. Three numbers, the third of which being 12, form a decreasing G.P. If third term were 9 instead of 12, the numbers would have formed an A.P. Then the common ratio of the original G.P. is

(a) $\frac{3}{7}$ (b) $\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

3. If n arithmetic means $a_1, a_2, \ldots, a_n$ are inserted between 50 and 200 and n harmonic means $h_1, h_2, \ldots, h_n$ are inserted between the same two numbers, then a_2h_{n-1} is equal to

(a) 500 (b) $\frac{10000}{n}$ (c) 10000 (d) $\frac{250}{n}$

4. Let $a_1, a_2, a_3, \ldots$ be a sequence such that $a_1 = 2$ and $a_n - a_{n-1} = 2n$ for all $n \geq 2$ then $a_1 + a_2 + a_3 + \ldots + a_{20}$ is

(a) 3040 (b) 3020 (c) 2020 (d) 3080

5. If $\sin A = a\cos B$ and $\cos A = b\sin B$, then $(a^2 - 1)\tan^2 A + (1 - b^2)\tan^2 B$ is equal to

(a) $\frac{a^2-b^2}{b^2}$ (b) $\frac{a^2-b^2}{a^2}$
(c) $\frac{a^2+b^2}{b^2}$ (d) $\frac{a^2+b^2}{a^2}$

6. $|\tan x + \sec x| = |\tan x| + |\sec x|$, $x \in [0, 2\pi]$ if and only if x belong to the interval

(a) $[0, \pi]$ (b) $\left[0, \frac{\pi}{2}\right[\cup \left]\frac{\pi}{2}, \pi\right]$
(c) $\left[\pi, \frac{3\pi}{2}\right[\cup \left]\frac{3\pi}{2}, 2\pi\right]$ (d) $]\pi, 2\pi]$

7. If $\tan\alpha = \frac{x^2-x}{x^2-x+1}$ and $\tan\beta = \frac{1}{2x^2-2x+1}$, $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha + \beta =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{4}$

8. If $\alpha + \beta - \gamma = \pi$ then $\sin^2\alpha + \sin^2\beta - \sin^2\gamma =$
(a) $2\sin\alpha\sin\beta\sin\gamma$ (b) $2\sin\alpha\sin\beta\cos\gamma$
(c) $2\cos\alpha\cos\beta\sin\gamma$ (d) $2\cos\alpha\cos\beta\cos\gamma$

9. If $\tan\theta = \sqrt{n}$ where $n \in N \geq 2$, $\sec 2\theta$ is always

(a) rational (b) irrational

(c) positive integer (d) none of these

10. If $\tan\beta = \cos\theta \cot\alpha$, then $\tan^2\frac{\theta}{2}$ is

(a) $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ (b) $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$

(c) $\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$ (d) $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

SECTION-II

Multiple Correct type

This section contains 7 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which **ONE OR MORE** is/are correct. [4 marks for correct answer and –1 for wrong answer]

11. The slope of a median, drawn from the vertex A of the triangle ABC is -2. The coordinate of the vertex B and C are respectively $(-1, 3)$ and $(3, 5)$. If the area of the triangle be 5 square units; then the distance of the vertex A from the origin is

(a) 6 (b) 4 (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

12. $A(1, 2)$ and $B(7, 10)$ are two fixed points. If $P(x, y)$ is such a point where $\angle APB = 60°$ and the area of the ΔAPB is maximum then the point P

(a) is on the line $3x + 4y = 36$

(b) is on any line perpendicular to AB

(c) is on the perpendicular bisector of AB

(d) is on the circle, passing through the points $(1, 2)$ and $(7, 10)$ and having radius 10

13. BC is the base of an isosceles triangle ABC. If the coordinates of B and C respectively are $(a + b, b - a)$ and $(a - b, a + b)$, the coordinate of A is

(a) (a, b) (b) (b, a)

(c) $\left(\frac{a}{b}, \frac{b}{a}\right)$ (d) $\left(1, \frac{b}{a}\right)$

14. If a, b and c are in H.P., then the value of $\frac{(ac+ab-bc)(ab+bc-ac)}{(abc)^2}$ is

(a) $\frac{(a+c)(3a-c)}{4a^2c^2}$ (b) $\frac{2}{bc}-\frac{1}{b^2}$

(c) $\frac{2}{bc}-\frac{1}{a^2}$ (c) $\frac{(a-c)(3a+c)}{4a^2c^2}$

15. If $\sqrt{2}\cos A = \cos B + \cos^3 B$ and $\sqrt{2}\sin A = \sin B - \sin^3 B$, then $\sin(A - B) =$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $-\frac{1}{2}$ (d) $-\frac{1}{3}$

SECTION-III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE OR MORE** is/are correct. [Correct answer : 4 marks, wrong answer :–1 mark]

Paragraph for 16. to Q. 18

The sum of the first and last term of a G.P. is 66, the product of first and last mean is 128 and sum of terms of the series is 126.

16. If the G.P. is increasing, then the number of terms is

(a) 9 (b) 8 (c) 12 (d) 6

17. If the G.P. is decreasing, then the sum of the series is

(a) 64 (b) 128 (c) 256 (d) 729

18. The difference between the greatest and least term is

(a) 78 (b) 126

(c) 128 (d) none of these

Paragraph for Q. 19 to Q. 21

If the roots of the equation $\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right) = 3\tan\frac{3\theta}{2}$ are $\alpha, \beta, \gamma, \delta$ and $\alpha \neq \beta \neq \gamma \neq \delta$, then

19. The value of $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}+\tan\frac{\delta}{2} =$

(a) $\frac{1}{3}$ (b) $\frac{8}{3}$ (c) $-\frac{8}{3}$ (d) 0

20. The value of $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}\tan\frac{\delta}{2}=$

(a) $-\frac{1}{3}$ (b) -2 (c) 0 (d) 7

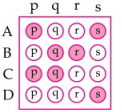
21. The value of $\frac{1}{\tan\frac{\alpha}{2}}+\frac{1}{\tan\frac{\beta}{2}}+\frac{1}{\tan\frac{\gamma}{2}}+\frac{1}{\tan\frac{\delta}{2}}=$

(a) -8 (b) 8 (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

SECTION-III

Matrix-Match Type

This section contains 3 questions.



Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example. If the correct matches are A-p and s ; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the above. [8 marks for correct answer and no negative marking for wrong answer and each row 2 marks]

22. Match the following:

	Column I		Column II
(A)	If a, b, c are in G.P., $\log_a 10$, $\log_b 10$, $\log_c 10$ are in	(p)	A.P.
(B)	If $\frac{a+be^x}{a-be^x}=\frac{b+ce^x}{b-ce^x}=\frac{c+de^x}{c-de^x}$ then a, b, c, d are in	(q)	H.P.
(C)	If a, b, c are in A.P. and a, x, b are in G.P., and b, y, c are in G.P. then x^2, b^2, y^2 are in	(r)	G.P.
(D)	If x, y, z are in G.P. and $a^x = b^y = c^z$ then $\log a, \log b, \log c$ are in	(s)	None of these

23. Match the following:

	Column I		Column II
(A)	$\cos(-1125°)$	(p)	1
(B)	The minimum value of $\sin\theta + \cos\theta$ in $0 < \theta < \frac{\pi}{2}$ is	(q)	$\frac{\sqrt{3}}{2}$
(C)	When n is even then the value of $\sin\left[n\pi+(-1)^n\frac{\pi}{3}\right]$	(r)	$\frac{1}{\sqrt{2}}$
(D)	$\cos510° \cdot \cos 330° + \sin 390° \cdot \cos120°=$	(s)	-1

24. Match the following:

	Column I		Column II
(A)	The distance between two points $(-1, a)$ and $(7, 4a)$ is 10 units. The value of a is	(p)	2
(B)	The area of the triangle formed by $(0, 0)$, $(a, 0)$ and $\left(\frac{1}{2}, a\right)$ is $\frac{1}{2}$ sq. unit. The value of a is	(q)	-2
(C)	The line $2x + y = k$ divides the line segment joining the points $(1, 1)$ and $(2, 4)$ in the ratio 3 : 2. The value of k is	(r)	1
(D)	If $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$, and $C(0, a)$ are collinear, then the value of $\frac{t_1+t_2}{t_1t_2}$	(s)	6

SOLUTIONS

PAPER-I

1. (b) : Here $s_n = cn(n-1) \therefore t_n = s_n - s_{n-1}$
$= cn(n-1) - c(n-1)(n-2)$
$= c(n-1)(n-n+2)$
$= 2c(n-1)$
So, $t_n = 2c(n-1) \Rightarrow t_n^2 = 4c^2(n-1)^2$
$\Rightarrow t_n^2 = 4c^2(n^2-2n+1)$
$\Rightarrow \sum t_n^2 = 4c^2\left[\sum n^2 - 2\sum n + n\right]$

$$= 4c^2\left[\frac{n(n+1)(2n+1)}{6} - \frac{2n(n+1)}{2} + n\right]$$

$$= 4c^2 n\left[\frac{2n^2+3n+1}{6} - (n+1) + 1\right]$$

$$= 4c^2 n\left[\frac{2n^2+3n+1-6n-6+6}{6}\right]$$

$$= \frac{4c^2 n}{6}[2n^2 - 3n + 1]$$

$$\sum t_n^2 = \frac{2}{3}c^2 n(2n-1)(n-1)$$

2. (a) : $a + b + c = xb$

$\Rightarrow$ $a + ar + ar^2 = x \cdot ar$

[let the common ratio of the G.P. be r]

$\Rightarrow$ $1 + (1 - x)r + r^2 = 0$

$\because$ r is real $\quad \therefore$ $(1-x)^2 - 4 \cdot 1 \cdot 1 > 0$

$\Rightarrow$ $x^2 - 2x + 1 - 4 > 0$

$\Rightarrow$ $x^2 - 2x - 3 > 0$

$\Rightarrow$ $(x + 1)(x - 3) > 0$

$\Rightarrow$ $x < -1$ or $x > 3$

3. (a) : Let $a = \frac{1}{k-d}$, $b = \frac{1}{k}$ and $c = \frac{1}{k+d}$

So, that a, b, c are in H.P.

Also given that a^2, b^2, c^2 are in A.P.

$\therefore$ $\frac{1}{(k-d)^2} + \frac{1}{(k+d)^2} = \frac{2}{k^2}$

$\Rightarrow$ $\frac{2(k^2+d^2)}{(k^2-d^2)^2} = \frac{2}{k^2}$

$\Rightarrow$ $k^2(k^2 + d^2) = k^4 - 2k^2d^2 + d^4$

$\Rightarrow$ $k^4 + k^2d^2 = k^4 - 2k^2d^2 + d^4 \Rightarrow d^4 - 3k^2d^2 = 0$

$\Rightarrow$ $d^2(d^2 - 3k^2) = 0, \because d \neq 0 \therefore d^2 - 3k^2 = 0$

$\Rightarrow$ $d^2 = 3k^2 \Rightarrow d = \pm\sqrt{3}k$

$\therefore$ $a = \frac{1}{k-\sqrt{3}k}$ or $\frac{1}{k+\sqrt{3}k}$, $b = \frac{1}{k}$ and

$c = \frac{1}{k+\sqrt{3}k}$ or $\frac{1}{k-\sqrt{3}k}$

$\therefore$ $a : b : c = \frac{1}{k-\sqrt{3}k} : \frac{1}{k} : \frac{1}{k+\sqrt{3}k}$

$$= \frac{1}{1-\sqrt{3}} : 1 : \frac{1}{1+\sqrt{3}}$$

$$= \frac{1+\sqrt{3}}{-2} : 1 : \frac{1-\sqrt{3}}{-2}$$

$a : b : c = (1+\sqrt{3}) : -2 : (1-\sqrt{3})$

OR

$a : b : c = \frac{1}{k+\sqrt{3}k} : \frac{1}{k} : \frac{1}{k-\sqrt{3}k}$

$= 1-\sqrt{3} : -2 : 1+\sqrt{3}$

4. (c): $f(x) = \cos[\pi]x + \cos[\pi x]$

$\therefore$ $f\left(\frac{\pi}{2}\right) = \cos[\pi]\frac{\pi}{2}\cos\left[\frac{\pi^2}{2}\right]$

$= \cos\frac{3\pi}{2} + \cos 4 \quad [\because \pi = 3.14 \ \therefore [\pi] = 3]$

$= 0 + \cos 4$

$\Rightarrow$ $f\left(\frac{\pi}{2}\right) = \cos 4$

5. (a) : $\because$ $c > \frac{3\pi}{4} \quad \therefore \quad 0 < A + B < \frac{\pi}{4}$

$\Rightarrow$ $0 < \tan(A+B) < \frac{\pi}{4}$

$\Rightarrow$ $\tan A + \tan B < 1 - \tan A \tan B$

$\Rightarrow$ $1 + \tan A + \tan B + \tan A \tan B < 2$

$\Rightarrow$ $(1 + \tan A)(1 + \tan B) < 2$

Also $\tan A > 0$; $\tan B > 0$

$\therefore$ $(1 + \tan A)(1 + \tan B) > 1$

$\therefore$ $1 < (1 + \tan A)(1 + \tan B) < 2$ is the required interval.

6. (a) : $a\sin x + b\{\cos(x+\theta) + \cos(x-\theta)\} = d$

$\Rightarrow$ $a\sin x + b\{\cos(x+\theta) + \cos(x-\theta)\} = d$

$\Rightarrow$ $a\sin x + 2b\cos x\cos\theta = d$

$$\frac{a}{\sqrt{a^2 + 4b^2\cos^2\theta}}\sin x + \frac{2b}{\sqrt{a^2 + 4b^2\cos^2\theta}}\cos x = \frac{d}{\sqrt{a^2 + 4b^2\cos^2\theta}} \quad \text{...(i)}$$

Let $\frac{a}{\sqrt{a^2 + 4b^2\cos^2\theta}} = \cos\alpha$ and

$\frac{2b}{\sqrt{a^2 + 4b^2\cos^2\theta}} = \sin\alpha$

$\therefore$ (i) becomes

$$\sin x\cos\alpha + \cos x\sin\alpha = \frac{d}{\sqrt{a^2 + 4b^2\cos^2\theta}}$$

$\Rightarrow \quad \sin(x+\alpha) = \dfrac{d}{\sqrt{a^2+4b^2\cos^2\theta}}$

$\therefore \quad -1 \le \dfrac{d}{\sqrt{a^2+4b^2\cos^2\theta}} \le 1$

$\Rightarrow \quad \left|\dfrac{d}{\sqrt{a^2+4b^2\cos^2\theta}}\right| \le 1$

$\Rightarrow \quad |d| \le \sqrt{a^2+4b^2\cos^2\theta}$

$\Rightarrow \quad d^2 \le a^2 + 4b^2\cos^2\theta$

$\Rightarrow \quad \dfrac{d^2-a^2}{4b^2} \le \cos^2\theta$

$\Rightarrow \quad \cos^2\theta \ge \dfrac{d^2-a^2}{4b^2}$

$\Rightarrow \quad |\cos\theta| \ge \dfrac{\sqrt{d^2-a^2}}{2|b|}$

7. (d) : Let $A = \theta$, $B = 2\theta$ and $C = 4\theta$

$\therefore \quad A + B + C = 7\theta = \pi$ ...(i)

Let $x = \tan\theta\tan2\theta + \tan2\theta\tan4\theta + \tan4\theta\tan\theta$

$= \tan A\tan B + \tan B\tan C + \tan C\tan A$

$= \sum \tan A\tan B$

But we know-

$\cos(A+B+C) = \cos A\cos B\cos C(1 - \Sigma\tan A\tan B]$

$\Rightarrow \quad \cos\pi = \cos\theta\cos2\theta\cos4\theta\,(1-x)$

$\Rightarrow \quad -1 = \dfrac{1}{2\sin\theta}\,[(2\sin\theta\cos\theta)\cos2\theta\cos4\theta(1-x)$

$= \dfrac{1}{2^2\sin\theta}\,[(2\sin2\theta\cos2\theta)\cos4\theta\,(1-x)]$

$= \dfrac{1}{2^3\sin\theta}\,[2\sin4\theta\cos4\theta)\,(1-x)]$

$= \dfrac{\sin8\theta}{8\sin\theta}\,(1-x) = \dfrac{\sin(7\theta+\theta)}{8\sin\theta}\,(1-x)$

$= \dfrac{\sin(\pi+\theta)}{8\sin\theta}\,(1-x)$

$\Rightarrow \quad -1 = -\dfrac{1}{8}(1-x) \Rightarrow (1-x) = 8 \;\therefore\; x = -7$

8. (a) : Let $x_1 = a$, $x_2 = ar$ and $x_3 = ar^2$

and $y_1 = b$, $y_2 = br$ and $y_3 = br^2$

$\therefore$ The vertices of the triangle are $A(a, b)$ $B(ar, br)$ and $C(ar^2, br^2)$

Now slope $AB = \dfrac{br-b}{ar-a} = \dfrac{b}{a}$

and slope of $BC = \dfrac{br^2-br}{ar^2-ar} = \dfrac{b}{a}$

Hence A, B, C are collinear.

9. (c): Let the point on y-axis be $(0, k)$

Now $(0+5)^2 + (k-4)^2 = (0-3)^2 + (k+2)^2$

$\Rightarrow \quad 25 + k^2 - 8k + 16 = 9 + k^2 + 4k + 4$

$\Rightarrow \quad 12k = 28 \;\Rightarrow\; k = \dfrac{7}{3}$

$\therefore$ The required point is $\left(0, \dfrac{7}{3}\right)$

10. (a) : $\dfrac{1}{2}\left|\{(p+1)(3-2p) + (2p+1)(2p-1) + (2p+2)(1-3)\}\right| = 9$

$\Rightarrow \quad \left|3p - 2p^2 + 3 - 2p + 4p^2 - 1 - 4p - 4\right| = 18$

$\Rightarrow \quad \left|\,2p^2 - 3p - 2\,\right| = 18$

$\Rightarrow \quad 2p^2 - 3p - 2 = 18 \quad$ or $\quad 2p^2 - 3p - 2 = -18$

when $2p^2 - 3p - 20 = 0$ then $(2p+5)(p-4) = 0$

$\therefore \quad p = 4, \;\dfrac{-5}{2}$

Also when $2p^2 - 3p + 16 = 0$. Then there is no real value of p.

11. (b) : $5x - y$, $2x + y$ and $x + 2y$ are in A.P.

$\therefore \quad 2(2x+y) = 5x - y + x + 2y$

$4x + 2y = 6x + y \;\Rightarrow\; 2x = y$ ...(i)

Again $(x-y)^2$, $(xy+1)$, $(y+1)^2$ are in G.P.

$\Rightarrow \quad (xy+1)^2 = (x-y)^2\,(y+1)^2$ ...(ii)

Putting $y = 2x$ in (ii), we get

$(2x^2+1)^2 = (-x)^2\,(2x+1)^2$

$\Rightarrow \quad 4x^4 + 4x^2 + 1 = x^2\,(4x^2 + 4x + 1)$

$\Rightarrow \quad 4x^4 + 4x^2 + 1 = 4x^4 + 4x^3 + x^2$

$\Rightarrow \quad 4x^3 - 3x^2 - 1 = 0$

$\Rightarrow \quad 4x^3 - 4x^2 + x^2 - x + x - 1 = 0$

$\Rightarrow \quad 4x^2(x-1) + x(x-1) + 1\,(x-1) = 0$

$(x-1)(4x^2 + x + 1) = 0$

$\therefore \quad x = 1 \quad \because 4x^2 + x + 1 = 0$ is not satisfied by any real value of x.

$\therefore \quad$ When $x = 1$, then from (i) we get $y = 2$

So, $x + y = 1 + 2 = 3$

12. (a, d) : $3(\cos2\phi - \cos2\theta) = 1 - \cos2\theta\cos2\phi$

$\therefore \quad \dfrac{1-\cos 2\theta\cos 2\phi}{\cos 2\phi-\cos 2\theta}=3$

or $\dfrac{1-\cos 2\theta\cos 2\phi-\cos 2\phi+\cos 2\theta}{1-\cos 2\theta\cos 2\phi+\cos 2\phi-\cos 2\theta}=\dfrac{3-1}{3+1}$

or $\dfrac{(1+\cos 2\theta)-\cos 2\phi(1+\cos 2\theta)}{(1-\cos 2\theta)+\cos 2\phi(1-\cos 2\theta)}=\dfrac{2}{4}$

or $\dfrac{(1+\cos 2\theta)(1-\cos 2\phi)}{(1-\cos 2\theta)(1+\cos 2\phi)}=\dfrac{1}{2}$

or $2\tan^2\phi=\tan^2\theta$ or $\tan\theta=\pm\sqrt{2}\tan\phi$

13. (a, b, c, d) :

$f_n(\theta)=\tan\left(\dfrac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)$ ...$(1+\sec 2^n\theta)$

$=\tan\left(\dfrac{\theta}{2}\right)\dfrac{(1+\cos\theta)}{\cos\theta}(1+\sec 2\theta)(1+\sec 4\theta)$ ...$(1+\sec 2^n\theta)$

$=\dfrac{\sin\dfrac{\theta}{2}}{\cos\dfrac{\theta}{2}}\cdot\dfrac{2\cos^2\dfrac{\theta}{2}}{\cos\theta}\cdot\dfrac{1+\cos 2\theta}{\cos 2\theta}(1+\sec 4\theta)$ ...$(1+\sec 2^n\theta)$

$=\dfrac{\left(2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right).2\cos^2\theta}{\cos\theta\cos 2\theta}(1+\sec 4\theta)$ ...$(1+\sec 2^n\theta)$

$=\dfrac{2\sin\theta\cos\theta}{\cos 2\theta}(1+\sec 4\theta)....(1+\sec 2^n\theta)$

$=\tan 2\theta\,(1+\sec 4\theta)...(1+\sec 2^n\theta)$

$=\tan 2^2\theta\,(1+\sec 2^3\theta)...(1+\sec 2^n\theta)$

.....

$=\tan 2^{n-1}\theta\,(1+\sec 2^n\theta)$

$f_n(\theta)=\tan 2^n\theta$

Now

$f_2\left(\dfrac{\pi}{16}\right)=\tan 2^2\left(\dfrac{\pi}{16}\right)=\tan\dfrac{\pi}{4}=1$

$f_3\left(\dfrac{\pi}{32}\right)=\tan 2^3\left(\dfrac{\pi}{32}\right)=\tan\dfrac{\pi}{4}=1$

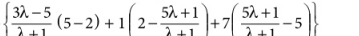
$f_4\left(\dfrac{\pi}{64}\right)=\tan 2^4\left(\dfrac{\pi}{64}\right)=\tan\dfrac{\pi}{4}=1$

$f_5\left(\dfrac{\pi}{128}\right)=\tan 2^5\left(\dfrac{\pi}{128}\right)=\tan\dfrac{\pi}{4}=1$

14. (a, c): Let P be (x, y)

$\therefore \quad x=\dfrac{3\lambda-5}{\lambda+1}$ and $y=\dfrac{5\lambda+1}{\lambda+1}$

$\therefore \quad P\equiv\left(\dfrac{3\lambda-5}{\lambda+1},\dfrac{5\lambda+1}{\lambda+1}\right)$



Now the area of the triangle PQR is

$\dfrac{1}{2}\left\{\dfrac{3\lambda-5}{\lambda+1}(5-2)+1\left(2-\dfrac{5\lambda+1}{\lambda+1}\right)+7\left(\dfrac{5\lambda+1}{\lambda+1}-5\right)\right\}$

$=\dfrac{1}{2}\left[\dfrac{3(3\lambda-5)+(2\lambda+2-5\lambda-1)+7(5\lambda+1-5\lambda-5)}{\lambda+1}\right]$

$=\dfrac{1}{2}\left(\dfrac{9\lambda-15+1-3\lambda-28}{\lambda+1}\right)=\dfrac{1}{2}\left(\dfrac{6\lambda-42}{\lambda+1}\right)$

$=\dfrac{3\lambda-21}{\lambda+1}$

by the problem $\left|\dfrac{3\lambda-21}{\lambda+1}\right|=2$

$\Rightarrow \quad \dfrac{3\lambda-21}{\lambda+1}=\pm 2$

when $\dfrac{3\lambda-21}{\lambda+1}=2$, then $\lambda=23$

when $\dfrac{3\lambda-21}{\lambda+1}=-2$, then $\lambda=\dfrac{19}{5}$

15. (a,c,d) : Centroid $=\left(\dfrac{x_1+x_2+x_3}{3},\dfrac{y_1+y_2+y_3}{3}\right)$

where (x_i, y_i), i = 1, 2, 3, are rational numbers. Centroid is a rational point. Orthocentre and circumcentre are also rational points.

16. (c): For any two positive unequal numbers G.M.^{2} = A.M. × H.M.

So the given statement-2 (reason) is wrong.

But $\dfrac{5}{2}\times\dfrac{8}{5}=4=2^2$

i.e. A.M. × H.M. = G.M.^{2}, so the statement-1 (Assertion) is true.

17. (d) : $1^3-2^3+3^3-4^3+....+11^3$

$=\dfrac{(-1)^{n-1}(4n^3+6n^2-1)-1}{8}$

$=\dfrac{(-1)^{10}(4\cdot 11^3+6\cdot 11^2-1)-1}{8}=756$

Remember :

$1^3-2^3+3^3$..... to n terms

$$= \frac{(-1)^{n-1}(4n^3 + 6n^2 - 1) - 1}{8}$$

Thus the statement – 1 is wrong for any odd integer,

$n \geq 1$, $n^3 - (n-1)^3 + ... + (-1)^{n-1} \cdot 1^3 = \frac{1}{4}(2n-1) \cdot (n+1)^2$ is true.

For example, say $n = 5$, then

L.H.S $= 5^3 - 4^3 + 3^3 - 2^3 + 1^3 = 125 - 64 + 27 - 8 + 1 = 81$

R.H.S $= \frac{1}{4}(2\times 5 - 1)(5+1)^2 = 81$

$\therefore$ LHS = RHS

18. (d)

19. (a) : $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$

$\Rightarrow 2\cos(\beta - \gamma) + 2\cos(\gamma - \alpha) + 2\cos(\alpha - \beta) + 3 = 0$

$\Rightarrow \cos^2\alpha + \sin^2\alpha + \cos^2\beta + \sin^2\beta + \cos^2\gamma + \sin^2\gamma + 2\cos\beta\cos\alpha + 2\sin\beta\sin\gamma + 2\cos\gamma\cos\alpha + 2\sin\gamma\sin\alpha + 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta = 0$

$\Rightarrow (\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta + 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha) + (\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma + 2\sin\gamma\sin\alpha) = 0$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

$\therefore \cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

20. (b) : Centroid

$$\left(\frac{a\cos t + b\sin t + 1}{3}, \frac{a\sin t - b\cos t}{3}\right)$$

Let $x = \frac{a\cos t + 3\sin t + 1}{3}$ and $y = \frac{a\sin t - b\cos t}{3}$

$\therefore$ $a\cos t + b\sin t = 3x - 1$ ...(i)

$a\sin t - b\cos t = 3y$ ...(ii)

Squaring and adding (i) and (ii) we get

$(3x-1)^2 + (3y)^2 = a^2 + b^2$

21. (4) : $ar^4 = 2$ ...(i)

$a \cdot ar \cdot ar^2 ... ar^8 = 128k$ ...(ii)

$\Rightarrow a^9 \cdot r^{1+2+...+8} = 128k \Rightarrow a^9 \cdot r^{4\times 9} = 128k$

$\Rightarrow (ar^4)^9 = 128k$

$\Rightarrow 2^9 = 128k$ (by (i))

$\Rightarrow 128 \times 4 = = 128k \Rightarrow k = 4$

22. (2) : Let the numbers be $\frac{a}{r}, a, ar$

$\therefore$ $\frac{a}{r} + a + ar = 14$ ...(i)

Now $\frac{a}{r} + 1, (a+1), ar - 1$ are in A.P.

$\therefore$ $2(a+1) = \frac{a}{r} + 1 + ar - 1 = \frac{a}{r} + ar$ ...(ii)

From (i) and (ii), we get

$a + 2(a+1) = 14 \Rightarrow 3a = 12$

$\Rightarrow a = 4$

From (i), $\frac{1}{r} + 1 + r = \frac{14}{a} = \frac{14}{4} = \frac{7}{2}$

$\Rightarrow \frac{1 + r + r^2}{r} = \frac{7}{2} \Rightarrow 2r^2 + 2r + 2 = 7r$

$\Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow (2r-1)(r-2) = 0$

$\Rightarrow r = 2, \frac{1}{2}$

23. (2) : If $A + B = 225°$, then $\cot(A+B) = \cot 225°$

$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = \cot 225° = \cot(180° + 45°)$

$= \cot 45° = 1$

$\Rightarrow \cot A \cot B - 1 = \cot A + \cot B$

$\Rightarrow \cot A \cot B = 1 + \cot A + \cot B$

$\Rightarrow 2\cot A \cot B = 1 + \cot A + \cot B + \cot A \cot B$

$\Rightarrow 2\cot A \cot B = (1 + \cot A)(1 + \cot B)$

$\therefore$ $\frac{1 + \cot A}{\cot A} \cdot \frac{1 + \cot B}{\cot B} = 2$

24. (0) : $x\sin\theta = y\sin\left(\theta + \frac{2\pi}{3}\right) = z\sin\left(\theta + \frac{4\pi}{3}\right) = \frac{1}{k}$ (say)

$\therefore$ $\frac{1}{x} = k\sin\theta$, $\frac{1}{y} = k\sin\left(\theta + \frac{2\pi}{3}\right)$,

$\frac{1}{z} = k\sin\left(\theta + \frac{4\pi}{3}\right)$

Now $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = k\left[(\sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right)\right]$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = k\left[\sin\theta - 2\sin(\pi + \theta)\cos\frac{\pi}{3}\right]$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = k[\sin\theta - \sin\theta]$

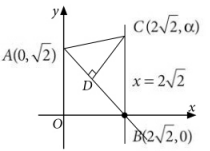
$\Rightarrow \frac{xy + yz + zx}{xyz} = 0 \Rightarrow xy + yz + zx = 0$

25. (5) : Let $A \equiv (0, \sqrt{2})$

$B \equiv (2\sqrt{2}, 0)$, and third vertex be $C \equiv (2\sqrt{2}, \alpha)$

Now $AB = \sqrt{(2\sqrt{2}-0)^2 + (0-\sqrt{2})^2}$

$AB = \sqrt{10}$



Now $AC = BC$

$\Rightarrow (2\sqrt{2})^2 + (\alpha - \sqrt{2})^2 = \alpha^2$

$\Rightarrow 8 + (\alpha - \sqrt{2})^2 = \alpha^2 \Rightarrow \alpha = \frac{5}{\sqrt{2}}$

Equation of AB is $\frac{x}{2\sqrt{2}} + \frac{y}{\sqrt{2}} = 1$

$\Rightarrow x + 2y = 2\sqrt{2}$

Now $CD = \frac{2\sqrt{2} + 2\times\frac{5}{\sqrt{2}} - 2\sqrt{2}}{\sqrt{1^2+2^2}} = \frac{\sqrt{2}\times 5}{\sqrt{5}} = \sqrt{10}$

$\therefore$ area of the ΔABC is $\frac{1}{2} AB \times CD$

$= \frac{1}{2}\times\sqrt{10}\times\sqrt{10} = \frac{1}{2}\times 10 = 5$ sq. units

PAPER-II

1. (b) : Given that a, b, c are in A.P.

$\therefore 2b = a + c$ ...(i)

But $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$

$\Rightarrow \frac{2b+b}{3} \geq \sqrt[3]{64} \Rightarrow \frac{3b}{3} \geq 4 \Rightarrow b \geq 4$

2. (b) : Let $\frac{12}{r^2}, \frac{12}{r}, 12$ be a decreasing G.P.

$\therefore \frac{12}{r^2}, \frac{12}{r}, 9$ are in A.P.

$\Rightarrow \frac{24}{r} = \frac{12}{r^2} + 9 \Rightarrow \frac{8}{r} = \frac{4}{r^2} + 3$

$\Rightarrow 3r^2 - 8r + 4 = 0 \Rightarrow (3r-2)(r-2) = 0$

$\Rightarrow r = 2, \frac{2}{3}$

$\because$ The given G.P. is decreasing one, $\therefore \quad r = \frac{2}{3}$

3. (c): $50, a_1, a_2, \ldots, a_n, 200$ are in A.P. ... (i)

$50, h_1, h_2, \ldots, h_n, 200$ are in H.P.

$\therefore \frac{1}{50}, \frac{1}{h_1}, \frac{1}{h_2}, \ldots \frac{1}{h_n}, \frac{1}{200}$ are in A.P.

Reversing $\frac{1}{200}, \frac{1}{h_n}, \frac{1}{h_{n-1}}, \ldots, \frac{1}{h_1}, \frac{1}{50}$ are in A.P.

Multiplying by 10000, we get

$50, \frac{10000}{h_n}, \frac{10000}{h_{n-1}}, \ldots \frac{10000}{h_1}, 200 \in$ A.P. ... (ii)

Thus (i) and (ii) are identical

$\Rightarrow a_2 h_{n-1} = 10{,}000$

4. (d) : Given that $a_1 = 2$, $a_n - a_{n-1} = 2n \ \forall n \geq 2$

$\therefore \quad a_n = 2n + a_{n-1}$

$a_2 = 4 + a_1 = 6$

$a_3 = 6 + a_2 = 12$

$a_4 = 8 + 12 = 20$

$a_5 = 10 + 20 = 35$

Now

$S = 2 + 6 + 12 + 20 + \ldots + a_n$

$S = \quad 2 + 6 + 12 + \ldots + a_{n-1} + a_n$

$\quad\quad (-)\ (-)\ (-)\ (-)\ (-)\ (-)$

$0 = 2 + 4 + 6 + 8 + \ldots$ to n terms $- a_n$

$\therefore \quad a_n = \frac{n}{2}[2\cdot 2 + (n-1)\times 2]$

$\Rightarrow a_n = n(n+1)$

$\therefore \quad \sum a_n = \sum n^2 + \sum n$

$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right]$

$\Rightarrow S_n = \frac{n(n+1)(n+2)}{3}$

$\therefore \quad S_{20} = \frac{20\times 21\times 22}{3} = 20\times 7\times 22 = 3080$

5. (a) : Given that $\sin A = a\cos B$ ... (i)

and $\cos A = b\sin B$ ...(ii)

(i)^2 + (ii)^2 gives $a^2\cos^2 B + b^2\sin^2 B = 1$

$\Rightarrow a^2\cos^2 B + B^2(1 - \cos^2 B) = 1$

$\Rightarrow (a^2 - b^2)\cos^2 B = 1 - b^2$

$\therefore \quad \cos^2 B = \frac{1-b^2}{a^2-b^2}$

$\therefore \quad \sin^2 B = 1 - \cos^2 B = \left(1 - \frac{1-b^2}{a^2-b^2}\right)$

$\Rightarrow \sin^2 B = \dfrac{a^2-1}{a^2-b^2} \quad \therefore \quad \tan^2 B = \dfrac{a^2-1}{1-b^2}$

Again (i) ÷ (ii) gives

$\tan A = \dfrac{a}{b}\cot B \Rightarrow \tan^2 A = \dfrac{a^2}{b^2}\cot^2 B$

$\Rightarrow \tan^2 A = \dfrac{a^2}{b^2}\cdot\dfrac{1-b^2}{a^2-1}$

Now $(a^2-1)\tan^2 A + (1-b^2)\tan^2 B$

$= (a^2-1)\dfrac{a^2}{b^2}\cdot\dfrac{1-b^2}{a^2-1} + (1-b^2)\cdot\dfrac{a^2-1}{1-b^2}$

$= \dfrac{a^2}{b^2}(1-b^2) + a^2 - 1 = \dfrac{a^2-b^2}{b^2}$

6. (b) : $|\sec x + \tan x| = |\sec x| + |\tan x|$

If sec$x$ and tan$x$ both have same sign

$\Rightarrow \sec x \tan x \geq 0 \Rightarrow \dfrac{\sin x}{\cos^2 x} \geq 0$

$\Rightarrow \sin x \geq 0$ but $\cos x \neq 0$

$\Rightarrow x \in \left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$

7. (a) : Given that $\tan\alpha = \dfrac{x^2-x}{x^2-x+1}$

$\Rightarrow \tan\alpha = \dfrac{K}{K+1}$ [Let $x^2 - x = K$]

$\tan\beta = \dfrac{1}{2x^2-2x+1} \Rightarrow \tan\beta = \dfrac{1}{2K+1}$

Now $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \dfrac{\dfrac{K}{K+1}+\dfrac{1}{2K+1}}{1-\dfrac{K}{K+1}\cdot\dfrac{1}{2K+1}} = \dfrac{K(2K+1)+K+1}{(K+1)(2K+1)-K}$

$= \dfrac{2K^2+2K+1}{2K^2+2K+1}$

$\Rightarrow \tan(\alpha+\beta) = 1$

8. (b) : Given that $\alpha + \beta - \gamma = \pi \quad \therefore \alpha - \gamma = \pi - \beta$

Now $\sin^2\alpha + \sin^2\beta - \sin^2\gamma$

$= \sin(\alpha+\gamma)\sin(\alpha-\gamma) + \sin^2\beta$

$= \sin(\alpha+\gamma)\sin(\pi-\beta) + \sin^2\beta$

$= \sin(\alpha+\gamma)\sin\beta + \sin^2\beta$

$= \sin\beta[\sin(\alpha+\gamma) + \sin\beta]$

$= \sin\beta[\sin(\alpha+\gamma) + \sin(\alpha-\gamma)]$

$= \sin\beta[2\sin\alpha\cos\gamma]$

9. (a) : Given that $\tan\theta = \sqrt{n} \quad \therefore \cos 2\theta = \dfrac{1-n}{1+n}$

$\therefore \sec 2\theta = \dfrac{1+n}{1-n} \qquad (n \in N \geq 2)$

sec2θ = rational number

10. (d) : Given that $\tan\beta = \cos\theta\cot\alpha$

$\Rightarrow \cos\theta = \dfrac{\tan\beta}{\cot\alpha} = \dfrac{\sin\beta}{\cos\beta}\cdot\dfrac{\sin\alpha}{\cos\alpha}$

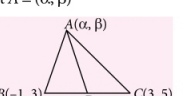
$\Rightarrow \dfrac{1-\tan^2\dfrac{\theta}{2}}{1+\tan^2\dfrac{\theta}{2}} = \dfrac{\sin\alpha\ \sin\beta}{\cos\alpha\ \cos\beta}$

$\Rightarrow \dfrac{\left(1+\tan^2\dfrac{\theta}{2}\right)-\left(1-\tan^2\dfrac{\theta}{2}\right)}{\left(1+\tan^2\dfrac{\theta}{2}\right)+\left(1-\tan^2\dfrac{\theta}{2}\right)}$

$= \dfrac{\cos\alpha\cos\beta - \sin\alpha\ \sin\beta}{\cos\alpha\cos\beta + \sin\alpha\ \sin\beta}$

$\Rightarrow \tan^2\dfrac{\theta}{2} = \dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

11. (a, c): Let $A \equiv (\alpha, \beta)$



Since AD is median, therefore D will be the mid-point of BC.

Hence $D \equiv (1, 4)$

by the problem $\dfrac{4-\beta}{1-\alpha} = -2$

$\Rightarrow 2\alpha + \beta = 6$ ... (i)

Again $\dfrac{1}{2}\{\alpha(3-5) - 1(5-\beta) + 3(\beta-3)\} = \pm 5$

$\Rightarrow -2\alpha + 4\beta = 24$ ... (ii)

or

$-2\alpha + 4\beta = 4$ ...(iii)

Solving (i) and (ii), we get $\alpha = 0$ and $\beta = 6$

$\therefore \quad (\alpha, \beta) = (0, 6)$

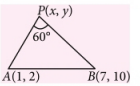
Solving (i) and (iii), we get $\alpha = 2$ and $\beta = 2$

$\therefore \quad (\alpha, \beta) = (2, 2)$

$\therefore$ distance of A from the origin is either 6 or $2\sqrt{2}$.

12. (a, c): Slope of $AP = \dfrac{y-2}{x-1}$

Slope of $BP = \dfrac{y-10}{x-7}$



Now, $\dfrac{\dfrac{y-2}{x-1} - \dfrac{y-10}{x-7}}{1 + \dfrac{(y-2)(y-10)}{(x-1)(x-7)}} = \pm\sqrt{3}$

$$\Rightarrow \quad \frac{4x-3y+2}{(x-1)(x-7)+(y-2)(y-10)} = \pm\frac{\sqrt{3}}{2} \qquad \text{... (i)}$$

Let area of the triangle be Δ

$$\therefore \quad \Delta = \left|\frac{1}{2}\{x(2-10)+1(10-y)+7(y-2)\}\right|$$

$$= \left|\frac{1}{2}(-8x+10-y+7y-14)\right|$$

$$= |-(4x-3y+2)|$$

Now from (i), we get

$$\Delta = \pm\frac{\sqrt{3}}{2}\{(x-1)(x-7)+(y-2)(y-10)\}$$

$$= \pm\frac{\sqrt{3}}{2}\{(x-4)^2+(y-6)^2-25\}$$

Clearly Δ will be maximum when $x = 4$ and $y = 6$

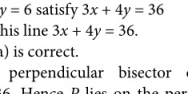
Since $x = 4$, $y = 6$ satisfy $3x + 4y = 36$

$\therefore$ P is on this line $3x + 4y = 36$.

So, option (a) is correct.

Again the perpendicular bisector of AB is $3x + 4y = 36$. Hence P lies on the perpendicular bisector of AB.

13. (d) : Let A be (x, y) then $AB = AC$, as



ΔABC is isosceles

$$\therefore \quad AB^2 = AC^2$$

$$\Rightarrow \quad \{x-(a+b)\}^2 + (y-(b-a))^2 = (x-(a-b))^2 + \{y-(a+b)\}^2$$

$$\Rightarrow \quad x^2 - 2x(a+b) + (a+b)^2 + y^2 - 2y(b-a) + (b-a)^2 = x^2 - 2x(a-b) + (a-b)^2 + y^2 - 2y(a+b) + (a+b)^2$$

$$\Rightarrow \quad -2ax - 2bx - 2by + 2ay = -2ax + 2bx - 2ay - 2by$$

$$\Rightarrow \quad 4ay = 4bx \Rightarrow ay = bx$$

The option (d) is correct but option (a), satisfies the above relation, but (a, b) is the mid-point of BC. It can't be (a).

14. (a, b) : a, b, c are in H.P.

$$\therefore \quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

$$\therefore \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \qquad \text{... (i)}$$

Now $\dfrac{(ac+ab-bc)(ab+bc-ac)}{(abc)^2}$

$$= \frac{ac+ab-bc}{abc} \cdot \frac{bc+ab-ac}{abc}$$

$$= \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right) \qquad \text{... (ii)}$$

$$= \left(\frac{1}{b}+\frac{1}{c}-\frac{2}{b}+\frac{1}{c}\right)\left(\frac{2}{b}-\frac{1}{b}\right) \qquad \text{[by (i)]}$$

$$= \left(\frac{2}{c}-\frac{1}{b}\right)\frac{1}{b} = \frac{2}{bc} - \frac{1}{b^2}$$

Also from (ii), we get

$$\left\{\frac{1}{2}\left(\frac{1}{a}+\frac{1}{c}\right)+\frac{1}{c}-\frac{1}{a}\right\}\left\{\frac{1}{c}+\frac{1}{a}-\frac{1}{2}\left(\frac{1}{a}+\frac{1}{c}\right)\right\}$$

$$= \left(\frac{1}{2a}+\frac{1}{2c}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{2}\left(\frac{1}{c}+\frac{1}{a}\right)\right) \quad \left[\because \frac{2}{b} = \frac{1}{a}+\frac{1}{c}\right]$$

$$= \left(\frac{3}{2}\cdot\frac{1}{c}-\frac{1}{a}\right)\left(\frac{a+c}{2ac}\right) = \frac{(3a-c)(a+c)}{4a^2c^2}$$

15. (b, d) : $(\sqrt{2}\cos A)^2 + (\sqrt{2}\sin A)^2$

$= (\cos B + \cos^3 B)^2 + (\sin B - \sin^3 B)^2$

$$\Rightarrow \quad 2 = \cos^2 B(1+\cos^2 B)^2 + \sin^2 B(1-\sin^2 B)^2$$

$$= \cos^2 B(1 + 2\cos^2 B + \cos^4 B) + \sin^2 B\cos^4 B$$

$$= \cos^2 B(1 + 2\cos^2 B) + \cos^4 B(\cos^2 B + \sin^2 B)$$

$$= \cos^2 B + 2\cos^4 B + \cos^4 B$$

$$2 = \cos^2 B + 3\cos^4 B$$

$$\Rightarrow \quad 3\cos^4 B + \cos^2 B - 2 = 0$$

$$\Rightarrow \quad (3\cos^2 B - 2)(\cos^2 B + 1) = 0$$

$$\therefore \quad \cos^2 B + 1 \neq 0$$

$$\therefore 3\cos^2 B - 2 = 0 \Rightarrow \cos^2 B = \frac{2}{3}$$

Again $\sqrt{2}\sin A\cos B - \sqrt{2}\cos A\sin B$

$= (\sin B - \sin^3 B)\cos B - (\cos B + \cos^3 B)\sin B$

$= -\sin^3 B\cos B - \cos^3 B\sin B$

$= -\sin B\cos B(\sin^2 B + \cos^2 B)$

$= -\sin B\cos B$

$\Rightarrow \sqrt{2}\sin(A-B) = -\sin B\cos B$

$\Rightarrow 2\sin^2(A-B) = \sin^2 B\cos^2 B$

$= (1-\cos^2 B)\cos^2 B$

$= \left(1-\frac{2}{3}\right)\cdot\frac{2}{3} = \frac{2}{9}$

$\therefore \sin^2(A-B) = \frac{1}{9} \Rightarrow \sin(A-B) = \pm\frac{1}{3}$

16. (d) : Given $a + ar^{n+1} = 66$ ... (i)

$ar \,.\, ar^n = 128$ ... (ii)

$S_{n+2} = 126$ ... (iii)

From (i) $a(1 + r^{n+1}) = 66$

$\Rightarrow a\left(1+\frac{128}{a^2}\right) = 66$ (by (ii))

$\Rightarrow a\left(\frac{a^2+128}{a^2}\right) = 66$

$\Rightarrow a^2 - 66a + 128 = 0$

$\Rightarrow a^2 - 64a - 2a + 128 = 0$

$\Rightarrow a(a-64) - 2(a-64) = 0$

$\Rightarrow (a-2)(a-64) = 0 \quad \therefore \quad a = 2, 64$

When $a = 2$, then from (ii) $r^{n+1} = 32 = 2^5$

Now from (iii), we get

$$\frac{2(r^{n+2}-1)}{r-1} = 126$$

$\Rightarrow r^{n+1}\,.\,r - 1 = 63(r-1)$

$\Rightarrow 32r - 1 = 63r - 63$

$\Rightarrow 31r = 62 \Rightarrow r = 2$

$\therefore \; r^{n+1} = 2^5$ gives $n = 4$

$\therefore$ number of terms $= n + 2 = 6$

when $a = 64$, then r becomes fraction and then G.P. becomes decreasing.

17. (b) : The G.P. is decreasing when $a = 64,\ r = \frac{1}{2}$

When $a = 64$, then from (ii)

$$r^{n+1} = \frac{128}{64\times 64} = \frac{1}{32} = \left(\frac{1}{2}\right)^5$$

$$r = \left(\frac{1}{2}\right)^{\frac{5}{n+1}} = \left(\frac{1}{2}\right)^{\frac{5}{4+1}} = \frac{1}{2}$$

$$\therefore \text{ Sum} = \frac{64\left(1-\frac{1}{2^6}\right)}{1-\frac{1}{2}} = 64\times 2\times\frac{63}{64} = 126$$

Note : The option (b) is wrong.

18. (d) : The smallest term is 2 and the greatest term is $2\cdot 2^5 = 64$

$\therefore$ The required difference is 62.

19. (d) : $\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right) = 3\tan\frac{3\theta}{2}$

$$\Rightarrow \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = 3\cdot\frac{3\tan\frac{\theta}{2}-\tan^3\frac{\theta}{2}}{1-3\tan^2\frac{\theta}{2}}$$

$$\Rightarrow 3\left(3\tan\frac{\theta}{2}-\tan^3\frac{\theta}{2}\right)\left(1-\tan\frac{\theta}{2}\right) = \left(1+\tan\frac{\theta}{2}\right)\left(1-3\tan^2\frac{\theta}{2}\right)$$

$$\Rightarrow 9\tan\frac{\theta}{2} - 3\tan^3\frac{\theta}{2} - 9\tan^2\frac{\theta}{2} + 3\tan^4\frac{\theta}{2} = 1+\tan\frac{\theta}{2} - 3\tan^2\frac{\theta}{2} - 3\tan^3\frac{\theta}{2}$$

$$\Rightarrow 3\tan^4\frac{\theta}{2} - 6\tan^2\frac{\theta}{2} + 8\tan\frac{\theta}{2} - 1 = 0$$

$\because$ $\alpha, \beta, \gamma, \delta$ are the roots of the above equation

$$\therefore \quad \tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}+\tan\frac{\delta}{2} = 0$$

20. (a) : $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}\cdot\tan\frac{\gamma}{2}\cdot\tan\frac{\delta}{2} = -\frac{1}{3}$

21. (b) :

$$\frac{\frac{1}{\tan\frac{\alpha}{2}}+\frac{1}{\tan\frac{\beta}{2}}+\frac{1}{\tan\frac{\gamma}{2}}+\frac{1}{\tan\frac{\delta}{2}}}{\frac{\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\cdot\tan\frac{\delta}{2} + \tan\frac{\alpha}{2}\cdot\tan\frac{\gamma}{2}\tan\frac{\delta}{2}+\tan\frac{\beta}{2}\cdot\tan\frac{\gamma}{2}\tan\frac{\delta}{2}}{\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}\cdot\tan\frac{\delta}{2}}}$$

$$= \frac{-8}{-1} = 8$$

Remember : If α, β, γ and δ be the roots of the biquadratic $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$, then

$$\alpha+\beta+\gamma+\delta = -\frac{a_1}{a_0}$$

$$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{a_2}{a_0}$$

$$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = -\frac{a_3}{a_0}$$

$$\alpha\beta\gamma\delta = \frac{a_4}{a_0}$$

22. A → q, B → r, C → p, D → r

(A) Given that a, b, c are in G.P.

$\therefore \quad b^2 = ac$

Now $\dfrac{1}{\log_a 10} + \dfrac{1}{\log_c 10} = \log_{10} a + \log_{10} c$

$= \log_{10} ac = \log_{10} b^2 \qquad [\because b^2 = ac]$

$= 2\log_{10} b = \dfrac{2}{\log_b 10}$

$\therefore \quad \dfrac{1}{\log_a 10} + \dfrac{1}{\log_c 10} = \dfrac{2}{\log_b 10}$

$\therefore \quad \log_a 10, \log_b 10, \log_c 10$ are in H.P.

(B) $\dfrac{a+be^x}{a-be^x} = \dfrac{b+ce^x}{b-ce^x}$

$\Rightarrow \quad \dfrac{a+be^x+a-be^x}{a+be^x-a+be^x} = \dfrac{b+ce^x+b-ce^x}{b+ce^x-b+ce^x}$

$\Rightarrow \quad \dfrac{2a}{2be^x} = \dfrac{2b}{2ce^x}$

$\Rightarrow \quad b^2 = ac \qquad \therefore \quad a, b, c,$ are in G.P.

Similarly, $\dfrac{b+ce^x}{b-ce^x} = \dfrac{c+de^x}{c-de^x}$

$\Rightarrow \quad b, c, d$ are in G.P.

$\therefore \quad a, b, c, d$ are in G.P.

(C) $a, b, c \in$ A.P. $\therefore \quad 2b = a + c$

$a, x, b \in$ G.P. $\qquad \therefore \quad x^2 = ab$

$b, y, c \in$ G.P. $\qquad \therefore \quad y^2 = bc$

Now $x^2 + y^2 = b(a + c) = b(2b) = 2b^2$

$\therefore \quad x^2, b^2, y^2$ are in A.P.

(D) Do it by yourself

23. A → r, B → p, C → q, D → s

(A) $\cos(-1125°) = \cos(1125°)$

$= \cos(12 \times 90° + 45°)$

$= \cos 45° = \dfrac{1}{\sqrt{2}}$

(B) $\sin\theta + \cos\theta$

$= \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\cos\theta + \dfrac{1}{\sqrt{2}}\sin\theta\right) = \sqrt{2}\cos\left(\theta - \dfrac{\pi}{4}\right)$

$= \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 1$ when $\theta = \dfrac{\pi}{2}$

(C) $\sin\left[n\pi + (-1)^n \dfrac{\pi}{3}\right] = \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

(D) $\cos 510° \cdot \cos 330° + \sin 390° \cdot \sin 120°$

$= \cos(5 \times 90° + 60°)\cos(4 \times 90° - 30°)$

$+ \sin(4 \times 90° + 30°) \times \left(-\dfrac{1}{2}\right)$

$= (-\sin 60°)(\cos 30°) - \dfrac{1}{2}\sin 30°$

$= -\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{4} = -\left(\dfrac{3}{4} + \dfrac{1}{4}\right) = -1$

24. A → p, q; B → q, s; C → t; D → p

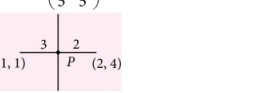
(A) By the problem $(-1-7)^2 + (a - 4a)^2 = 100$

$\Rightarrow \quad 9a^2 + 64 = 100 \Rightarrow \quad a^2 = 4 \Rightarrow \quad 4 \Rightarrow \quad a = \pm 2$

(B) $\dfrac{1}{2}\left|\{0(0-a) + a(a-0) + \dfrac{1}{2}(0)\}\right| = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{2}|a^2| = \dfrac{1}{2} \Rightarrow \quad a^2 = 1 \quad \therefore \quad a = \pm 1$

(C) Co-ordinate of P is $\left(\dfrac{8}{5}, \dfrac{14}{5}\right)$



$\because \quad \left(\dfrac{8}{5}, \dfrac{14}{5}\right)$ lies on $2x + y = K$

$\therefore \quad 2\left(\dfrac{8}{5}\right) + \dfrac{14}{5} = K \quad \therefore \quad K = 6$

(D) $at_1^2(2at_2 - a) + at_2^2(a - 2at_1) + 0 = 0$

$\Rightarrow \quad t_1^2(2t_2 - 1) + t_2^2(1 - 2t_1) = 0$

$\Rightarrow \quad 2t_1^2t_2 - t_1^2 + t_2^2 - 2t_1t_2^2 = 0$

$\Rightarrow \quad 2t_1t_2(t_1 - t_2) = t_1^2 - t_2^2 = (t_1 + t_2)(t_1 - t_2)$

$\therefore \quad \dfrac{t_1 + t_2}{t_1t_2} = 2$

Contd. from page no. 26

12.

- $2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) & \text{, if } -1 \le x \le 1 \\ \pi - \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) & \text{, if } x > 1 \\ -\pi - \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) & \text{, if } x < -1 \end{cases}$

- $2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) & \text{, if } 0 \le x < \infty \\ -\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right) & \text{, if } -\infty < x \le 0 \end{cases}$

13.

- $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\dfrac{x}{\sqrt{1-x^2}} = \cot^{-1}\dfrac{\sqrt{1-x^2}}{x} = \sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right) = \text{cosec}^{-1}\left(\dfrac{1}{x}\right)$

- $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right) = \cot^{-1}\dfrac{x}{\sqrt{1-x^2}} = \sec^{-1}\dfrac{1}{x} = \text{cosec}^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

- $\tan^{-1}x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\dfrac{1}{x}\right) = \sec^{-1}\sqrt{1+x^2} = \text{cosec}^{-1}\left(\dfrac{\sqrt{1+x^2}}{x}\right)$

14.

- If $x_1, x_2, \ldots, x_n \in R$, then

$$\tan^{-1}x_1 + \tan^{-1}x_2 + \ldots + \tan^{-1}x_n = \tan^{-1}\left(\frac{S_1 - S_3 + S_5 - S_7 + \ldots}{1 - S_2 + S_4 - S_6 + \ldots}\right)$$

where S_k = Sum of the products of $x_1, x_2, \ldots, x_n$ taken k at a time.

VERY SHORT ANSWER TYPE (1 MARK)

1. Evaluate : $\sin\left[\dfrac{\pi}{3} - \sin^{-1}\left(\dfrac{-1}{2}\right)\right]$.

2. Find the principal value of $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$.

3. Find the principal value of $\sin^{-1}\left(\sin\dfrac{4\pi}{5}\right)$.

4. Using principal value, evaluate $\sin^{-1}\left(\sin\dfrac{3\pi}{5}\right)$

5. Solve for x : $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \dfrac{1}{2}\tan^{-1}x$, $x > 0$.

6. Find the principle value of $\tan^{-1}(\sqrt{3})$.

7. Show that $\tan^{-1}\left\{\tan\dfrac{5\pi}{6}\right\} \neq \dfrac{5\pi}{6}$ and find its value.

SHORT ANSWER TYPE (4 MARKS)

8. Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2.$$

9. Prove that :

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$

10. Solve for x : $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\,x)$

11. Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right).$$

12. Prove that : $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

13. Prove that :

$$\tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\}=\frac{\pi}{4}-\frac{x}{2},\ \text{if}\ \pi<x<\frac{3\pi}{2}$$

14. Prove that :

$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$$

15. Write $\cot^{-1}(\sqrt{1+x^2}-x)$ in the simplest form.

SOLUTIONS

1. $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)=\sin\left(\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right)$

$[\because \sin^{-1}(-\theta)=-\sin^{-1}\theta]$

$=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\frac{\pi}{2}=1$

2. $\cos^{-1}\frac{\sqrt{3}}{2}=\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)=\frac{\pi}{6},\ \frac{\pi}{6}\in[0,\pi]$

3. $\sin\left(\frac{4\pi}{5}\right)=\sin\left(\pi-\frac{\pi}{5}\right)=\sin\frac{\pi}{5}$ and

$\frac{\pi}{5}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Therefore, principal value of $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)=\frac{\pi}{5}$.

4. $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{5}\right)\right]$

$=\sin^{-1}\left[\sin\left(\frac{2\pi}{5}\right)\right]=\frac{2\pi}{5},\ \frac{2\pi}{5}\in\left[\frac{-\pi}{2},\ \frac{\pi}{2}\right]$

5. We have, $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,\ (x>0)$

$\Rightarrow \tan^{-1}1-\tan^{-1}x=\frac{1}{2}\tan^{-1}x$

$\Rightarrow \frac{3}{2}\tan^{-1}x=\tan^{-1}1=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}x=\frac{\pi}{4}\times\frac{2}{3}=\frac{\pi}{6}\Rightarrow x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

$\Rightarrow x=\frac{1}{\sqrt{3}}$

6. Let $\tan^{-1}(\sqrt{3})=\theta$, then

$\tan\theta=\sqrt{3}=\tan\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3}\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

7. $\tan^{-1}\left\{\tan\frac{5\pi}{6}\right\}=\tan^{-1}\left\{\tan\left(\pi-\frac{\pi}{6}\right)\right\}$

$\tan^{-1}\left\{\tan\left(\frac{-\pi}{6}\right)\right\}\quad[\because \tan(\pi-\theta)=\tan(-\theta)]$

$=-\frac{\pi}{6}\quad\left[\because -\frac{\pi}{6}\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)\right]$

$\therefore\quad \tan^{-1}\left\{\tan\frac{5\pi}{6}\right\}=-\frac{\pi}{6}$

8. Putting $x^2=\cos2\theta$, we get

$$\text{L.H.S.}=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{1+\cos2\theta}+\sqrt{1-\cos2\theta}}{\sqrt{1+\cos2\theta}-\sqrt{1-\cos2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

[dividing numerator and denominator by $\cos\theta$]

$$=\tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\theta\right)\right\}=\left(\frac{\pi}{4}+\theta\right)$$

$$=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2=\text{R.H.S}$$

9. $\text{L.H.S.}=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}$

$$=\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{1}{5}\right)}{1-\frac{1}{2}\times\frac{1}{5}}\right)+\tan^{-1}\frac{1}{8}$$

$$=\tan^{-1}\left(\frac{5+2}{10-1}\right)+\tan^{-1}\frac{1}{8}=\tan^{-1}\frac{7}{9}+\tan^{-1}\frac{1}{8}$$

$$=\tan^{-1}\left[\frac{\left(\frac{7}{9}+\frac{1}{8}\right)}{\left(1-\frac{7}{9}\times\frac{1}{8}\right)}\right]=\tan^{-1}\left(\frac{56+9}{72-7}\right)$$

$$=\tan^{-1}\left(\frac{65}{65}\right)=\tan^{-1}1=\frac{\pi}{4}=\text{R.H.S.}$$

10. We have, $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(2\operatorname{cosec} x\right)$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x$$

$$\Rightarrow \cos x = \sin x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}.$$

11. Let $x = \cos^{-1}\frac{12}{13}$ and $y = \sin^{-1}\frac{3}{5}$

or $\cos x = \frac{12}{13}$ and $\sin y = \frac{3}{5}$

Now, $\sin x = \sqrt{1-\cos^2 x}$ and $\cos y = \sqrt{1-\sin^2 y}$

$$\Rightarrow \sin x = \sqrt{1-\frac{144}{169}} \text{ and } \cos y = \sqrt{1-\frac{9}{25}}$$

$$\Rightarrow \sin x = \frac{5}{13} \text{ and } \cos y = \frac{4}{5}$$

We know that,

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$= \frac{5}{13}\times\frac{4}{5} + \frac{12}{13}\times\frac{3}{5} = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

$$\Rightarrow x + y = \sin^{-1}\left(\frac{56}{65}\right)$$

12. L.H.S. $= 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\left[\frac{\frac{2}{3}}{1-\frac{1}{9}}\right] + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left[\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right]$$

$$= \tan^{-1}\left(\frac{21+4}{28-3}\right) = \tan^{-1}\frac{25}{25}$$

$$= \tan^{-1}1 = \frac{\pi}{4} = \text{R.H.S.}$$

13. L.H.S. $= \tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\}$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\frac{x}{2}}+\sqrt{2\sin^2\frac{x}{2}}}{\sqrt{2\cos^2\frac{x}{2}}-\sqrt{2\sin^2\frac{x}{2}}}\right\}$$

$$= \tan^{-1}\left\{\frac{-\cos\frac{x}{2}+\sin\frac{x}{2}}{-\cos\frac{x}{2}-\sin\frac{x}{2}}\right\}$$

$$\left[\because\ \pi < x < \frac{3\pi}{2}\ \therefore\ \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}\right.$$

$$\left.\therefore\ \left|\cos\frac{x}{2}\right| = -\cos\frac{x}{2}, \left|\sin\frac{x}{2}\right| = \sin\frac{x}{2}\right]$$

$$= \tan^{-1}\left\{\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right\} = \tan^{-1}\left\{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\} = \frac{\pi}{4}-\frac{x}{2} = \text{R.H.S.}$$

$$\left[\because \pi < x < \frac{3\pi}{2}\ \therefore -\frac{\pi}{2} < \frac{\pi}{4}-\frac{x}{2} < -\frac{\pi}{4}\right]$$

14. Let $\cos^{-1}\left(\frac{a}{b}\right) = \theta$. Then, $\cos\theta = \frac{a}{b}$

Now, L.H.S. $= \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$$= \frac{1+\tan(\theta/2)}{1-\tan(\theta/2)} + \frac{1-\tan(\theta/2)}{1+\tan(\theta/2)}$$

$$= \frac{(1+\tan(\theta/2))^2 + (1-\tan(\theta/2))^2}{1-\tan^2(\theta/2)}$$

$$= 2\left(\frac{1+\tan^2(\theta/2)}{1-\tan^2(\theta/2)}\right) = \frac{2}{\cos\theta} = \frac{2b}{a} = \text{R.H.S}$$

15. Putting $x = \cot\theta$, we get

$$\cot^{-1}(\sqrt{1+x^2}-x) = \cot^{-1}(\operatorname{cosec}\theta - \cot\theta)$$

$$= \cot^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \cot^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \cot^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right] = \frac{\pi}{2}-\frac{\theta}{2} = \left(\frac{\pi}{2}-\frac{1}{2}\cot^{-1}x\right)$$

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Q.1 If the sides BC, CA, AB of a triangle ABC are in A.P., then find the value of $\tan\frac{A}{2}\tan\frac{C}{2}$.

– Yati Khurana, Punjab

Ans. In usual notation

$$\tan\frac{A}{2}\tan\frac{C}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}\cdot\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \frac{s-b}{s} = \frac{c+a-b}{a+b+c} = \frac{2b-b}{2b+b} = \frac{1}{3}$$

Q.2 Let a sequence $x_1, x_2, x_3, \ldots$ of complex numbers be defined by

$x_1 = 0$ and $x_{n+1} = x_n^2 - i$ for $n \geq 1$,

where $i = \sqrt{-1}$. Then find the distance between x_{1994} and origin.

– Ramesh Kr, Patna

Ans. By the given relations

$x_2 = x_1^2 - i = -i$;

$x_3 = x_2^2 - i = (i)^2 - i = -1 - i = -(1 + i)$

Find $x_4, x_5, \ldots$.similarly. In general, we see that if $x_n = -(1 + i)$ then

$x_{n+1} = x_n^2 - i$

$= (1 + i)^2 - i = 1 + 2i + (-1) - i = i$;

$x_{n+2} = x_{n+1}^2 - i$

$= i^2 - i = -1 - i = -(1 + i)$

It follows that $x_3 = x_5 = \ldots\ldots x_{1993} = -(1 + i)$.

and $x_2 = x_4 = \ldots\ldots x_{1994} = i$

Hence $|x_{1994}| = 1$

Q.3 When $x + x^{1998} + x^{1999} + x^{2000} + x^{2001}$ is divided by $x^3 - x$, find the remainder.

– Chandan Biswas, Warangal

Ans. The remainder should be a polynomial of degree less than or equal to 2. So, let the remainder be $ax^2 + bx + c$.

Let $F(x) = x + x^{1998} + x^{1999} + x^{2000} + x^{2001}$

$= (x)(x + 1)(x - 1)Q(x) + ax^2 + bx + c$.

where $Q(x)$ is a polynomial of degree 1998.

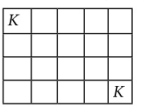
$F(0) = 0 = c$; $F(1) = 5 = a + b$; $F(-1) = -1 = a - b$

$\therefore \quad a = 2, b = 3$ and $c = 0$.

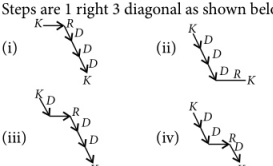
The remainder is $2x^2 + 3x$.

Q.4 In one move the king can go to any adjacent square, along a row, column or diagonal. How many routes with the minimum number of moves are there for the king to travel from the top left square to the bottom right square?

– Shwetha V. Thrissur



Ans. There are 4 rows and 5 columns in the board. Taking the diagonal step king moves down as well as right. He needs 1 right and 3 diagonal steps. Steps are 1 right 3 diagonal as shown below :



Thus there are four ways with 4 moves and 4 moves is the least.

Q.5 A polygonal prism has 2010 edges. Find the number of the faces of the prism.

– Md. Irfan, Azamgarh

Ans. Let the polygon constructing the prism be 'n' sides. Then two polygonal faces will have $2n$ edges.

Then n vertices of corresponding polygons are connected by $2n$ (vertical) edges to the n vertical faces. Hence, a polygonal prism constructed from a polygon of n sides will have $(n + 2)$ faces and $3n$ edges.

$\therefore \quad 3n = 2010 \Rightarrow n = 670$ and the number of faces $670 + 2 = 672$. ■■

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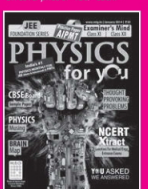

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